Semi-cognitive angle estimation for adaptive array radars

Michał Meller

PIT-RADWAR S.A., ul. Hallera 233A, 80-502 Gdańsk, POLAND
Department of Automatic Control, Faculty of Electronics, Telecommunications and Computer Science,
Gdańsk University of Technology, ul. Narutowicza 11/12, 80-233 Gdańsk, POLAND
e-mail: michal.meller@pitradwar.com

***Abstract*—The problem of angle estimation in a transmit adaptive array radar is considered. Somewhat in contrast to previous contributions on cognitive radar, which emphasized achieving the best possible performance, the proposed approach aims at reaching just the required amount of performance. This frees radar resources for other tasks and improves overall efficacy of the system. Furthermore, performance specification is made on the plot, rather than track, level. The proposed solution employs a performance predictor and a computationally cheap optimization mechanism. Simulation experiments demonstrate that the desired level of performance is indeed achieved and that the controller makes reasonable decisions.**

***Index Terms*—cognitive radar, fully adaptive array, angle estimation, transmit adaptivity, optimization**

I. Introduction

When compared with current state of the art, first radars were obviously very simple devices in all aspects. In particular, they operated with a – more rather than less – fixed set of parameters. Tuning of these parameters was tasked to the operators, which meant that performance of the radar was very much dependent on the operator's level of skill and reaction time.

Obviously, development of radar occurred at a rapid pace. Important milestones were marked by introduction of monopulse operation, adoption of digital signal processing, development of Doppler filtering, introduction of automated detection and tracking, electronic scanning or active arrays, among others.

The human operator was no longer able to keep up with this growth of capabilities and the need for radar management arose. A specialized software module was given the tasks of radar scheduling, analyzing environment, choosing the best operating frequency and waveform, and so on.

However, this module alone cannot exploit full capabilities offered by modern radar arrays. This is because it lacks high level situational awareness, knowledge of performance requirements and mission goals. This is where the operator steps in, and aids the software by adjusting some of its settings. However, as the human is simply not capable of micromanagement on a timescale of milliseconds, the system still operates far from optimally.

Cognitive radar, whose origins can be attributed to Guerci [1] and Haykin [2], aims to reduce this capability gap by radically improving radar management. This approach emphasizes feedback between the receiver and the transmitter parts of the radar (Fig. 1). The cognitive controller works out its decisions using probabilistic information about targets, obtained from the tracker. These decisions are optimal in the sense of minimizing the cost function associated with radar's mission goals.

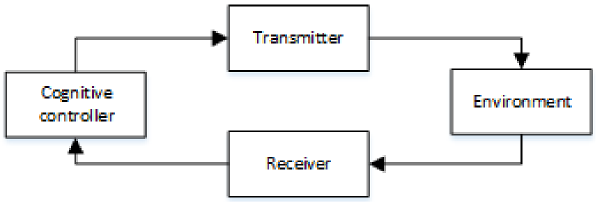

Figure 1. Simplified block diagram of a cognitive radar.

The concept received a lot of attention recently, and the research gained more momentum. Significant contributions include refinement and unification of theoretical foundations of cognitive radars with applications to tracking and detection [3], adaptive sequential Bayesian estimation [4], space-time code design [5], operation under interference [6], cognitive radar networks [7], [8] and implementation of cognitive radar [9], [10], among others.

While a lot of existing contributions tend to focus on range and velocity tracking [2], [3], [7], this paper considers application of cognition to angle estimation in a fully adaptive array. This direction is motivated by the fact that, at least in search radars, meeting requirements on angle estimation accuracy may be more difficult than meeting requirements on range estimation accuracy. We also take a step back and rewrite goals of the controller in such a way that it more consistent with industry standard, which often emphasizes performance on the plot, rather than track, level.

The paper is organized as follows. Section II presents the adopted structure of the radar, echo model and specifies assumptions and goals. Section III presents the solution to the performance prediction subproblem. Section IV presents an efficient solution to the optimization subproblem. Section V presents simulation results. Section VI concludes.

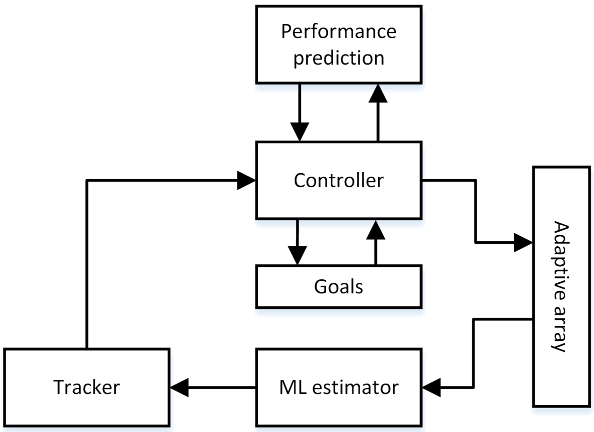

Figure 2. Radar system under consideration

II. Problem statement

A. Radar system under consideration

Block diagram of the radar system under consideration is depicted in Fig. 2. For simplicity, the parts which are outside of the scope of this paper, such as the detector, were omitted.

The so-called perception-action cycle [2] starts with the controller receiving information about expected target position, along with covariance matrices which characterize uncertainty of that expectation. It then attempts to find the action which will result in the desired accuracy of target angle estimates. To do this, it queries the performance prediction module with different values of parameters characterizing possible actions. These performance predictions are verified against goals and constraints and the optimal action is chosen and executed.

Upon reception of the echo, the radar performs maximum likelihood (ML) estimation of target angle and feeds this estimate to the tracker. The tracker updates its target's state estimates and delivers the results to the controller, which closes the loop.

B. Echo model, assumptions and goals

Suppose that the radar emits a trail of $N \geq 1$ pulses. Without loss of generality, we can assume that the pulses during dwell are not coherent. The coherent case can be handled by employing coherent integration of pulses and setting $N = 1$.

When dealing with angle estimation, multipath may have a significant impact on the accuracy of angle estimation [11]. We will assume that this phenomenon does not take place. In practice this means that we are dealing with azimuth estimation or that the target elevation is large enough for multipath to have negligible influence on the estimator's accuracy.

Under the above two assumptions, the echo signal received by the array during the n-th pulse, $n = 1, 2, \ldots, N$, can be modeled as

$$\boldsymbol{y}_n = AFr_n e^{j\phi_n} B_{\mathrm{TX}}(\boldsymbol{w}_{\mathrm{TX}}, \alpha)\boldsymbol{a}(\alpha) + \boldsymbol{v}_n \ , \tag{1}$$

where $\boldsymbol{y}_n$ denotes the echo signal vector, $\boldsymbol{v}_n$ is the measurement noise vector, α is the target's angle, $\boldsymbol{a}(\alpha)$ is the steering vector at the target,

$$B_{\mathrm{TX}}(\boldsymbol{w}_{\mathrm{TX}}, \alpha) = \boldsymbol{w}_{\mathrm{TX}}^H \boldsymbol{a}(\alpha) \tag{2}$$

is the (square root) transmit beampattern at the target resulting from the array distribution during transmit given by the vector $\boldsymbol{w}_{\mathrm{TX}}$ (normalized in such a way that magnitude each element of $\boldsymbol{w}_{\mathrm{TX}}$ does not exceed 1), r_n^2 and ϕ_n denote the target's radar cross section (RCS) and echo phase during the n-th pulse, F is a factor which covers propagation effects (including loss), while A is the amplitude of the transmitted pulse (that is, both A and $\boldsymbol{w}_{\mathrm{TX}}$ specify the parameters used for transmit

We will assume that RCS of the target is constant during dwell, $r_n \equiv r = \mathrm{const}$. Probability density function $p(\sigma)$ of the random variable $\sigma = r^2$ will be assumed to be known. Pulse phases ϕ_n, $n = 1, 2, \ldots, N$ are modeled as a sequence of i.i.d random variables with uniform distribution in the interval $[0, 2\pi]$. Target a'priori angle is characterized by its probability density function $p(\alpha)$. Finally, we assume that the noise vectors $\boldsymbol{v}_n$ form a zero mean, i.i.d. circular complex Gaussian distributed sequence with covariance matrix $\sigma_v^2 \boldsymbol{I}$, where $\boldsymbol{I}$ denotes the eye matrix of an appropriate size.

The controller's goal is to minimize the following quantity

$$Q = A^2 \boldsymbol{w}_{\mathrm{TX}}^{\mathrm{H}} \boldsymbol{R} \boldsymbol{w}_{\mathrm{TX}} \ , \tag{3}$$

where

$$\boldsymbol{R} = \int_{\Omega} \boldsymbol{a}(\alpha)\boldsymbol{a}^{\mathrm{H}}(\alpha) d\alpha \tag{4}$$

subject to

$$P_{\Delta\hat{\alpha}^2} < \Delta\alpha_{\max}^2 \ . \tag{5}$$

$$0 < A < A_{\max} \tag{6}$$

and each element of $\boldsymbol{w}_{\mathrm{TX}}$ having magnitude no larger than 1. The symbol Ω appearing in eq. (4) denotes the set of angles of interest (typically covering sidelobe region), while $P_{\Delta\hat{\alpha}^2}$ in eq. (5) is the predicted mean square error of the ML angle estimator.

C. Remarks

Before moving forward, let us makes a few remarks. First, note that our setup emphasizes reaching just the right level of performance. This is in contrast to classical contributions on cognitive radar, where the problem formulation usually focuses on reaching the best possible accuracy (see e.g. [2]). This allows us to trade excessive performance for gains in other areas – in case of cost criterion (3) we try to keep radar operation covert by keeping power radiated into sidelobes at a low level. Other design choices are, of course, possible – a reasonable option is to e.g. minimize time spent on the target, i.e. to minimize N.

Second, as evidenced by (5), we measure accuracy of the system at the output of the ML estimator. This arrangement also differs from the usual one, because it corresponds to the plot, rather than track, level. To motivate this choice, note that tracking has the potential for both improving and reducing system accuracy. The latter situation may occur when the assumed object motion model is poorly matched to actual behavior of the target. Due to this difficulty, industrial performance specifications often remove the influence of the tracker from the picture, and so are we.

It follows that, from the controllers' point of view, the tracker's main purpose will be to provide a'priori pdf of target's angle, so that an appropriate transmit beampattern can be selected. Because of this important difference, we refer to our approach as semi-cognitive.

Finally, we take note that the assumption that $p(\sigma)$ is known is somewhat unrealistic and is a clear weakness of the scheme in its present form. Practical solution should be extended with an on-line estimator of target RCS. Due to space limitations, this topic will not be covered in the paper.

III. Performance prediction

Due to differences between our problem formulation and the classical one, we cannot use posterior Cramér-Rao lower bound (PCRB) to predict performance of the estimator. This is because PCRB takes into account available prior information, i.e. it accounts for the presence of the tracker [12].

We cannot use standard Cramér-Rao lower bound (CRB) as well. This bound applies to estimation of unknown *deterministic* parameters, while in our case the parameters – angle α, echo strength r and phases ϕ_n, $n = 1,\ 2,\ \ldots,\ N$ – are all random quantities.

The right choice seems to be the expected Cramér-Rao bound (ECRB), which is obtained by averaging CRB over joint pdf of α, r and ϕ_n, $n = 1,\ 2,\ \ldots,\ N$

$$\mathbf{ECRB} = \mathrm{E}_{\boldsymbol{\theta}}\left[\mathbf{CRB}(\boldsymbol{\theta})\right] \ , \tag{7}$$

where $\boldsymbol{\theta} = [\alpha\ r\ \phi_1\ \phi_2\ \ldots \phi_N]^{\mathrm{T}}$ denotes the parameter vector.

The CRB is computed in a standard way, i.e as the inverse of the Fisher information matrix

$$\mathbf{CRB}(\boldsymbol{\theta}) = \boldsymbol{J}^{-1}(\boldsymbol{\theta}) \ , \tag{8}$$

where

$$\boldsymbol{J}(\boldsymbol{\theta}) = \mathrm{E}_{\boldsymbol{v}_1\ldots\boldsymbol{v}_N}\left[\left(\frac{\partial l}{\partial \boldsymbol{\theta}}\right)\left(\frac{\partial l}{\partial \boldsymbol{\theta}}\right)^{\mathrm{T}}\right] \ , \tag{9}$$

and l denotes the log-likelihood function of measurements.

Since we are dealing with the Gaussian case (recall that $\boldsymbol{v}_n$'s are assumed to be complex Gaussian distributed), the log-likelihood function takes the form

$$l = C + \frac{1}{\sigma_v^2}\sum_{n=1}^{N}\left(\boldsymbol{y}_n - \boldsymbol{m}_n(\boldsymbol{\theta})\right)^{\mathrm{H}}\left(\boldsymbol{y}_n - \boldsymbol{m}_n(\boldsymbol{\theta})\right) \ , \tag{10}$$

where C is a constant, whose exact form is not important, and

$$\boldsymbol{m}_n(\boldsymbol{\theta}) = AFre^{j\phi_n}B_{\mathrm{TX}}(\boldsymbol{w}_{\mathrm{TX}}, \alpha)\boldsymbol{a}(\alpha) \ . \tag{11}$$

Following [12], the Fisher information matrix can be computed as

$$\boldsymbol{J}(\boldsymbol{\theta}) = \frac{2}{\sigma_v^2}\sum_{n=1}^{N}\mathrm{Re}\left[\left(\frac{\partial \boldsymbol{m}_n^{\mathrm{H}}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)\left(\frac{\partial \boldsymbol{m}_n(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^{\mathrm{T}}}\right)\right] \ . \tag{12}$$

After performing necessary differentiations and cross-multiplying, one obtains

$$\boldsymbol{J}(\boldsymbol{\theta}) = \frac{2|AF|^2}{\sigma_v^2}\boldsymbol{J}_0(\boldsymbol{\theta}) \tag{13}$$

where

$$\boldsymbol{J}_0(\boldsymbol{\theta}) = \mathrm{Re}\begin{bmatrix} Nr^2Z_1 & NrZ_2^* & jr^2Z_2^* & jr^2Z_2^* & \ldots & jr^2Z_2^* \\ NrZ_2 & NZ_3 & jrZ_3^* & jrZ_3^* & \ldots & jrZ_3^* \\ -jr^2Z_2 & -jrZ_3 & r^2Z_3 & 0 & \ldots & 0 \\ -jr^2Z_2 & -jrZ_3 & 0 & r^2Z_3 & \ldots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ -jr^2Z_2 & -jrZ_3 & 0 & \ldots & 0 & r^2Z_3 \end{bmatrix} , \tag{14}$$

z^* denotes complex conjugate of z and

$$\begin{aligned} Z_1 &= T^{\mathrm{H}}T \\ Z_2 &= S^{\mathrm{H}}T \\ Z_3 &= S^{\mathrm{H}}S \\ S &= B_{\mathrm{TX}}(\boldsymbol{w}_{\mathrm{TX}}, \alpha)\boldsymbol{a}(\alpha) \\ T &= \frac{\partial B_{\mathrm{TX}}(\boldsymbol{w}_{\mathrm{TX}}, \alpha)}{\partial \alpha}\boldsymbol{a}(\alpha) + B_{\mathrm{TX}}(\boldsymbol{w}_{\mathrm{TX}}, \alpha)\frac{\partial \boldsymbol{a}(\alpha)}{\partial \alpha} \\ \frac{\partial B_{\mathrm{TX}}(\boldsymbol{w}_{\mathrm{TX}}, \alpha)}{\partial \alpha} &= \boldsymbol{w}_{\mathrm{TX}}^{\mathrm{H}}\frac{\partial \boldsymbol{a}(\alpha)}{\partial \alpha} \ . \end{aligned} \tag{15}$$

Observe that (14) depends on the transmit beampattern $B_{\mathrm{TX}}(\boldsymbol{w}_{\mathrm{TX}}, \alpha)$. This will play an important role during optimization, because it will make the inverse of the information matrix become "small" for those choices of $\boldsymbol{w}_{\mathrm{TX}}$ which are steered away from the target or whose beamwidths are not matched to uncertainty of target angle.

Computation of ECRB involves inverting (14) and then averaging over α and r (note that $\boldsymbol{J}_0$ does not depend on ϕ_n, $n = 1,\ 2,\ \ldots,\ N$). Fortunately, since we are interested only in the accuracy of angle estimation, this labor intensive task can be made somewhat simpler using the Schur complement method. Since Z_3 is a real valued quantity, $\boldsymbol{J}_0(\boldsymbol{\theta})$ takes the form

$$\boldsymbol{J}_0(\boldsymbol{\theta}) = \boldsymbol{J}_0(\alpha, r) = \mathrm{Re}\begin{bmatrix} Nr^2Z_1 & NrZ_2^* & jr^2Z_2^* & jr^2Z_2^* & \ldots & jr^2Z_2^* \\ NrZ_2 & NZ_3 & 0 & 0 & \ldots & 0 \\ -jr^2Z_2 & 0 & r^2Z_3 & 0 & \ldots & 0 \\ -jr^2Z_2 & 0 & 0 & r^2Z_3 & \ldots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ -jr^2Z_2 & 0 & 0 & \ldots & 0 & r^2Z_3 \end{bmatrix} . \tag{16}$$

It follows that

$$\mathbf{CRB}(\alpha, r) = \left(\boldsymbol{J}_{1,1} - \boldsymbol{J}_{1,\ 2:end}^{\mathrm{H}}\boldsymbol{J}_{2:end,\ 2:end}^{-1}\boldsymbol{J}_{2:end,1}\right)^{-1} , \tag{17}$$

where MATLAB-style notation was used to describe various submatrices of $\boldsymbol{J}(\boldsymbol{\theta})$. Taking into account that $\boldsymbol{J}_{2:end,\ 2:end}$ is a diagonal matrix, one can arrive at

$$\mathrm{CRB}(\alpha, r) = \frac{\sigma_v^2}{2|AF|^2 r^2} K_0^{-1}(\boldsymbol{w}_{\mathrm{T}X}, \alpha)$$
$$K_0(\boldsymbol{w}_{\mathrm{T}X}, \alpha) = \left(K_1 - \boldsymbol{K}_2^{\mathrm{T}} \boldsymbol{K}_3^{-1} \boldsymbol{K}_2\right)^{-1} \tag{18}$$

where

$$K_1 = NZ_1$$
$$\boldsymbol{K}_2 = \mathrm{Re}\left\{\left[\begin{array}{ccccc} NZ_2 & -jZ_2 & -jZ_2 & \dots & -jZ_2 \end{array}\right]^{\mathrm{H}}\right\}$$
$$\boldsymbol{K}_3 = \mathrm{diag}\left(NZ_3,\ Z_3,\ Z_3,\ \dots,\ Z_3\right) \tag{19}$$

Finally, after performing averaging over r and α, one obtains

$$\mathrm{ECRB} = \frac{\sigma_v^2}{2|AF|^2} \mathrm{E}_r\left[r^{-2}\right] \mathrm{E}_\alpha\left[K_0^{-1}(\boldsymbol{w}_{\mathrm{T}X}, \alpha)\right] \tag{20}$$

Note that the above formula replaces two-dimensional integration over joint pdf $p(r, \alpha)$ with two one-dimensional integrals over marginal distributions $p(r)$ and $p(\alpha)$ [or, as $r^2 = \sigma$, over $p(\sigma)$ and $p(\alpha)$].

IV. Optimization

Direct optimization of (3)-(5) may be demanding in terms of computational complexity. Suppose however, that a library $\mathcal{W}$ of prespecified transmit distributions is available. For each $\boldsymbol{w}_{\mathrm{T}X} \in \mathcal{W}$ one can minimize (3) subject to (5) with respect to A only, and then pick the optimal distribution by comparing the values of the cost function obtained during independent optimizations. This approach is attractive, because an explicit formula the optimal value of A for a given $\boldsymbol{w}_{\mathrm{T}X}$ can be found. Additionally, the search can be parallelized in a straightforward way.

To find A we first compute the two expectations appearing in (20), i.e. $\mathrm{E}_r\left[r^{-2}\right] = \mathrm{E}_\sigma\left[\sigma^{-1}\right]$ and $\mathrm{E}_\alpha\left[K_0^{-1}(\boldsymbol{w}_{\mathrm{T}X}, \alpha)\right]$. Note that the first expectation requires almost no effort when RCS fluctuations are modeled using chi-square distribution [13] or its generalization, the gamma distribution [14]. In both cases the result is a simple function of average target RCS. For gamma distributed RCS with shape parameter γ_s and rate parameter γ_r it holds that

$$\mathrm{E}[r^2] = \mathrm{E}[\sigma] = \frac{\gamma_s}{\gamma_r} = \sigma_{av}$$
$$\mathrm{E}[r^{-2}] = \mathrm{E}[\sigma^{-1}] = \frac{\gamma_r}{\gamma_s - 1} = \frac{1}{\sigma_{av} - \frac{1}{\gamma_r}}\ , \tag{21}$$

where σ_{av} denotes average target RCS.

Second, since it is obvious that the constraint (5) will be active for the optimal solution ()provided that the optimization problem is feasible) one immediately obtains

$$A^2 = \frac{\sigma_v^2}{2|F|^2 \Delta\alpha_{\max}^2} \mathrm{E}_r\left[r^{-2}\right] \mathrm{E}_\alpha\left[K_0^{-1}(\boldsymbol{w}_{\mathrm{T}X}, \alpha)\right]\ . \tag{22}$$

If the above obtained value of A^2 is smaller than $A_{\max}^2$ then the solution is feasible. Otherwise, it should be discarded, because it violates pulse amplitude constraint.

Evaluating (3) to find the associated cost is straightforward and needs no further comment. The optimal solution is the one which is feasible and minimizes (3). In case no entry from the library $\mathcal{W}$ is feasible, the choice must be based on another criterion, e.g. minimization of $ECRB$.

V. Simulation experiments

Consider a linear array with $M = 12$ elements, uniformly spaced at a distance $d = \lambda/4$, where λ denotes the wavelength. For convenience, the angle will be expressed using its sine, i.e. α is the sine of actual angle, $\alpha \in [-1, 1]$, and the steering vector takes the form

$$a(\alpha) = \left[1\ e^{j\frac{\pi}{2}\alpha}\ \dots\ e^{j\frac{\pi}{2}(M-1)\alpha}\right]^{\mathrm{T}}\ . \tag{23}$$

Without loss of generality, the measurement noise will have unit variance, $\sigma_v^2 = 1$. The library of transmit distributions $\mathcal{W}$ shall consist of six beampatterns, all steered towards boresight. The first beampattern employs uniform weighting, while the remaining 5 patterns employ Chebyshev weighting with sidelobe level equal to $-20,\ -25,\ -30,\ -35,\ -40$ dB, respectively.

The simulated target fluctuates according to Swerling III model with average RCS $\sigma_{av} = 2.5$ m^2. It follows that $\mathrm{E}[r^{-2}] = 0.8$ m^{-2}. For simplicity, we set the number of pulses to $N = 1$.

The target's prior distribution of angle is $\mathcal{N}(0, \sigma_\alpha^2)$, i.e. zero mean Gaussian with standard deviation equal to $\sigma_\alpha = 0.04$, or 40 millisines. The desired angle estimation accuracy is 3.5 milisines. The maximum pulse amplitude is $A_{\max} = 100$.

To simulate the effect of the distance between the radar and the target we simply set

$$|F|^2 = \frac{C}{R^4}\ , \tag{24}$$

where (without loss of generality) $C = 1$ and $R \in [R_{\min}, R_{\max}]$, where $R_{\min} = 1000$ m and $R_{\max} = 5000$ m.

Finally, the set of angles of interest in (4) is

$$\Omega = [-1,\ -b] \cup [b,\ 1]\ , \tag{25}$$

where $b = 0.3$.

Fig. 3 shows choices made by the controller for different values of distance R. Not unexpectedly, when the target is near, the sixth beam, i.e. the beam with the lowest sidelobe level, is selected. As the target moves away from the radar, the amplitude of the pulse is gradually increased to make up for the two-way propagation. Eventually, it is not possible to reach the desired accuracy with the sixth beam, because the required pulse amplitude is above $A_{\max}$. At this point the controller starts using beams with increasing sidelobe level, because these are able to illuminate the target stronger without exceeding $A_{\max}$. Finally, even the uniform beam does not allow one to reach the desired accuracy. However, as this choice leads to the best estimation accuracy, the controller still choses the uniform weighting.

In the next example we introduce only one alteration. The standard deviation of the target a'priori angle is increased

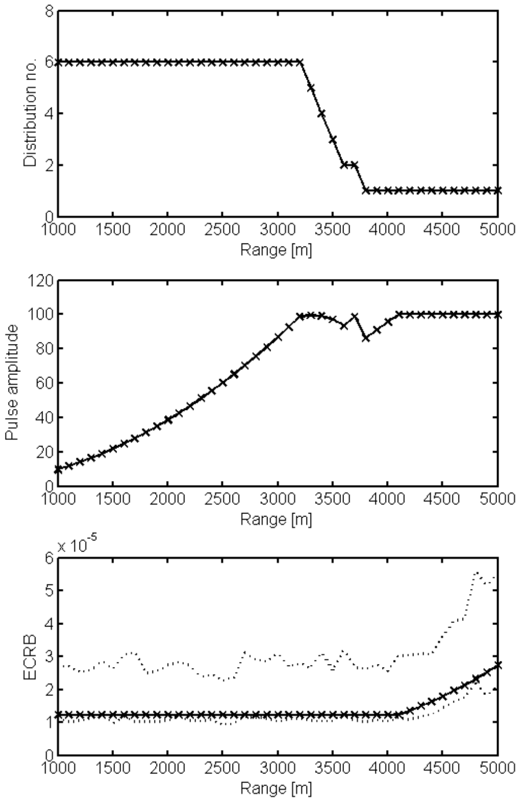

Figure 3. Results of the first simulation experiment. Top: Transmit distributions selected from the library for different ranges. Middle: Corresponding optimal pulse amplitudes. Bottom: Resulting values of $ECRB$ (solid line) compared with 75% and 90% quantiles of actual squared estimation error (dotted lines).

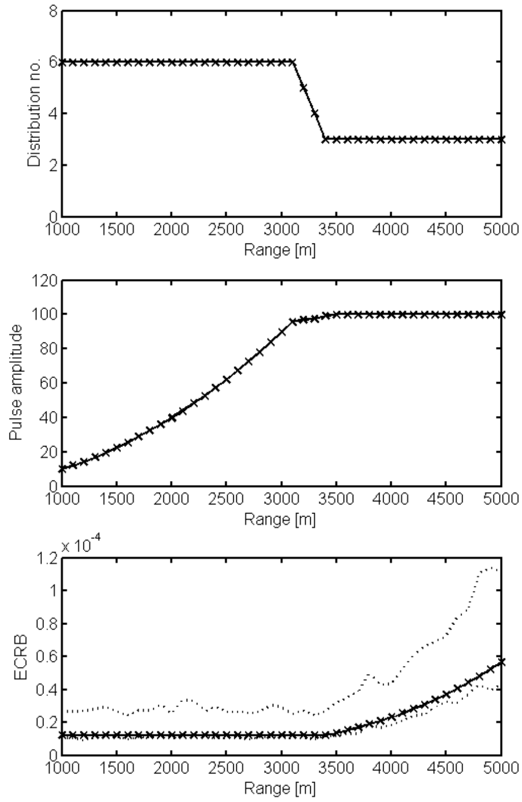

Figure 4. Results of the second simulation experiment. Top: Transmit distributions selected from the library for different ranges. Middle: Corresponding optimal pulse amplitudes. Bottom: Resulting values of $ECRB$ (solid line) compared with 75% and 90% quantiles of actual squared estimation error (dotted lines).

to 50 millisines. The resulting optimal choices made by the controller are shown in Fig. 4. Observe that the two first distributions are no longer optimal for large distances. This stems from the fact that these two distributions result in too narrow transmit beampatterns. As the uncertainty of target a'priori angle was increased, using these distributions will likely incur a large performance loss. The wider beams are therefore the better choice.

VI. Conclusions and Future Work

The problem of adapting the transmit parameters of the radar so as to achieve the desired angle estimation accuracy was considered. The solution employs a library of array distributions and a simple optimizer. Future work should consider extending the scheme with on-line target RCS estimator and applying convex optimization techniques to solve the problem.

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