Podstawy elektrotechniki

Teoria obwodów elektrycznych

Obwody prądu sinusoidalnego

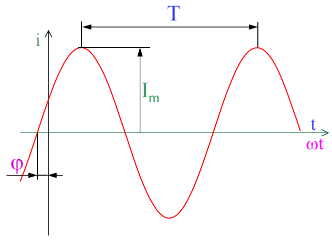
Parametry przebiegu sinusoidalnego

$$i(t) = I_m \sin(\omega t + \varphi)$$

Wartość chwilowa prądu

Amplituda prądu (wartość maksymalna)

faza



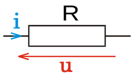
φ – faza początkowa

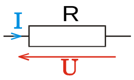
ω – pulsacja

$\omega = 2\pi f$ $\quad$ f – częstotliwość

$f = \dfrac{1}{T}$ $\quad$ T – okres

Wartość skuteczna





$i(t)$ – funkcja okresowa

T – okres funkcji

$I = \text{const.}$

$$E = RI^2T$$

$$E = \int_0^T Ri^2 dt = R\int_0^T i^2 dt \longrightarrow RI^2T = R\int_0^T i^2 dt$$

$$\boxed{I = \sqrt{\frac{1}{T}\int_0^T i^2 dt}}$$

Wartość skuteczna prądu okresowego

Wartość skuteczna

Wartość skuteczna prądu okresowego jest to wartość zastępczego prądu stałego, który w ciągu czasu T wywoła taki sam skutek cieplny.

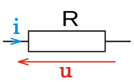
Dla prądu sinusoidalnego

$$I_{sk} = \sqrt{\frac{1}{T}\int_0^T [I_m \sin(\omega t)]^2 dt} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \frac{1-\cos(2\omega t)}{2} dt} = \frac{I_m}{\sqrt{2}}$$

Dla napięcia sinusoidalnego $\quad u(t) = U_m \sin(\omega t)$

$$U_{sk} = \frac{U_m}{\sqrt{2}}$$

Wartość średnia



$$I_{śr} = \frac{1}{T}\int_0^T i(t)dt$$

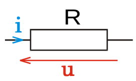
$$i(t) = I_m \sin(\omega t) \Longrightarrow I_{śr} = \frac{1}{T}\int_0^T I_m \sin(\omega t)dt = 0$$

Wartość średnia półokresowa

$$I_{śr} = \frac{2}{T}\int_0^{T/2} I_m \sin(\omega t)dt = \frac{2I_m}{T}\frac{1}{\omega}[-\cos(\omega t)]_0^{T/2} = \frac{2I_m}{\pi}$$

$$\omega = \frac{2\pi}{T}$$

Wartość średnia



Moc chwilowa

$$p(t) = u(t) \cdot i(t)$$

Energia pobrana w ciągu okresu

$$W = \int_0^T u \cdot i\, dt = \int_0^T p\, dt$$

Wartość średnia energii za okres

$$P = \frac{W}{T} = \frac{1}{T}\int_0^T p\, dt$$

Moc czynna prądu okresowego

Moc czynna P jest wartością średnią mocy chwilowej p(t)

Rezystor w obwodzie prądu sinusoidalnego

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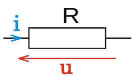

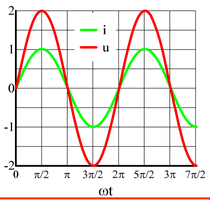

$$i(t) = I_m \sin(\omega t)$$

$$u(t) = R \cdot i(t) = RI_m \sin(\omega t) \qquad RI_m \rightarrow U_m$$

$$u(t) = U_m \sin(\omega t)$$

$$U_m = RI_m \quad \Big| \frac{1}{\sqrt{2}}$$

$$U = RI$$ - dla wartości skutecznych

Napięcie na rezystancji jest w fazie z prądem.

Cewka w obwodzie prądu sinusoidalnego

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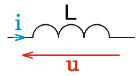

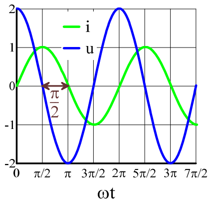

$$i(t) = I_m \sin(\omega t)$$

$$u(t) = L\frac{di}{dt} = L\omega I_m \cos(\omega t) \qquad L\omega I_m \rightarrow U_m$$

$$u(t) = U_m \cos(\omega t) = U_m \sin\left(\omega t + \tfrac{\pi}{2}\right)$$

$$U_m = \omega L I_m = X_L I_m \quad \Big| \frac{1}{\sqrt{2}}$$

X_L – reaktancja indukcyjna

$$U = X_L I$$ - dla wartości skutecznych

Napięcie na indukcyjności wyprzedza w fazie prąd o kąt π/2 (90°).

Kondensator w obwodzie prądu sinusoidalnego

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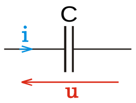
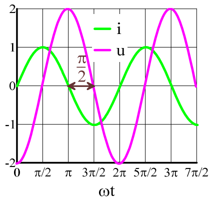


$$i(t) = I_m \sin(\omega t)$$

$$i = C\frac{du}{dt}$$



$$u(t) = \frac{1}{C}\int i dt = \frac{1}{C\omega} I_m [-\cos(\omega t)] \qquad \frac{1}{C\omega} I_m \rightarrow U_m$$

$$u(t) = -U_m \cos(\omega t) = U_m \sin\left(\omega t - \tfrac{\pi}{2}\right)$$

Napięcie na pojemności opóźnia się w fazie za prądem o kąt π/2 (90°).

$$U_m = \frac{1}{\omega C} I_m = X_C I_m \quad \Big| \frac{1}{\sqrt{2}}$$

X_C – reaktancja pojemnościowa

$$U = X_C I$$ - dla wartości skutecznych

Gałąź RL w obwodzie prądu sinusoidalnego

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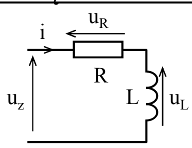


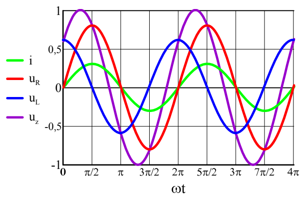
$$i(t) = I_m \sin \omega t$$

$$u_z = u_R + u_L$$

$$u_z = RI_m \sin \omega t + X_L I_m \cos \omega t$$

$$= I_m (R \sin \omega t + X_L \cos \omega t)$$

$$= I_m \sqrt{R^2 + X_L^2} \sin(\omega t + \varphi)$$



$$\varphi = \arctan \frac{X_L}{R}$$

$$0 < \varphi < 90°$$

$$u_z = U_m \sin(\omega t + \varphi)$$

$$U_m = I_m \sqrt{R^2 + X_L^2}$$

Gałąź RC w obwodzie prądu sinusoidalnego

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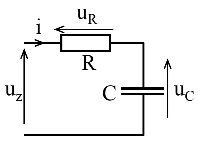


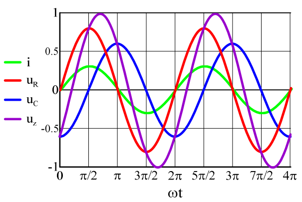
$$i(t) = I_m \sin \omega t$$

$$u_z = u_R + u_C$$

$$u_z = RI_m \sin \omega t - X_C I_m \cos \omega t$$

$$= I_m (R \sin \omega t - X_C \cos \omega t)$$

$$= I_m \sqrt{R^2 + X_C^2} \sin(\omega t + \varphi)$$



$$\varphi = \arctan \frac{-X_C}{R}$$

$$-90° < \varphi < 0$$

$$u_z = U_m \sin(\omega t + \varphi)$$

$$U_m = I_m \sqrt{R^2 + X_C^2}$$

Gałąź RLC w obwodzie prądu sinusoidalnego

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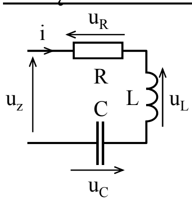
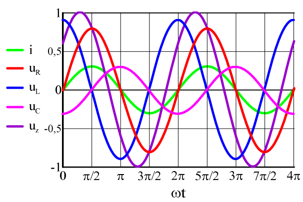


$$i(t) = I_m \sin \omega t$$

$$u_z = u_R + u_L + u_C$$

$$u_z = RI_m \sin \omega t + X_L I_m \cos \omega t - X_C I_m \cos \omega t$$

$$= I_m \sqrt{R^2 + (X_L - X_C)^2} \sin(\omega t + \varphi)$$



$$\varphi = \arctan \frac{X_L - X_C}{R}$$

$$-90° < \varphi < 90°$$

$$u_z = U_m \sin(\omega t + \varphi)$$

$$U_m = I_m \sqrt{R^2 + (X_L - X_C)^2}$$

Sumowanie wielkości sinusoidalnych

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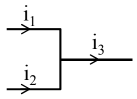
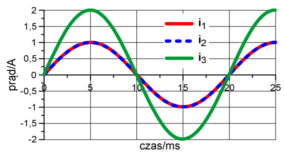


$$i_1(t) = 1\sin\omega t \qquad i_2(t) = 1\sin\omega t$$

$$i_3 = i_1 + i_2 = 1\sin\omega t + 1\sin\omega t = 2\sin\omega t$$

$$I_{1sk} = \frac{1}{\sqrt{2}} \approx 0,707 \qquad I_{2sk} = \frac{1}{\sqrt{2}} \approx 0,707 \qquad I_{3sk} = \frac{2}{\sqrt{2}} \approx 1,414$$

$$I_{1sk} + I_{2sk} = 0,707 + 0,707 = 1,414 = I_{3sk}$$



$$I_{1sk} + I_{2sk} = I_{3sk}$$

Sumowanie wielkości sinusoidalnych

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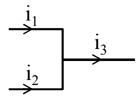

$$i_1(t) = 1\sin\omega t \qquad i_2(t) = 1\sin(\omega t - 60°)$$

$$i_3 = i_1 + i_2 = 1\sin\omega t + 1\sin(\omega t - 60°) =$$

$$1\sin\omega t + 1\sin(\omega t)\cos(-60°) + 1\cos(\omega t)\sin(-60°) =$$

$$\frac{3}{2}\sin\omega t - \frac{\sqrt{3}}{2}\cos(\omega t) =$$

$$\sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2}\,\sin\left(\omega t + \operatorname{arctg}\frac{-\sqrt{3}/2}{3/2}\right) = \sqrt{3}\sin(\omega t - 30°)$$

Sumowanie wielkości sinusoidalnych

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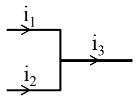
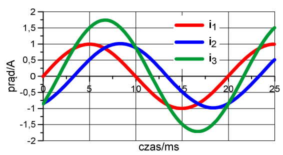


$$i_1(t) = 1\sin\omega t \qquad i_2(t) = 1\sin(\omega t - 60°)$$

$$i_3(t) = \sqrt{3}\sin(\omega t - 30°)$$

$$I_{1sk} = \frac{1}{\sqrt{2}} \approx 0,707 \qquad I_{2sk} = \frac{1}{\sqrt{2}} \approx 0,707 \qquad I_{3sk} = \frac{\sqrt{3}}{\sqrt{2}} \approx 1,225$$

$$I_{1sk} + I_{2sk} = 0,707 + 0,707 = 1,414$$



$$I_{1sk} + I_{2sk} \neq I_{3sk}$$

W ogólnym przypadku wartości skuteczne prądów i napięć nie spełniają praw Kirchhoffa

Liczby zespolone

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$\underline{z} = a + bi$ — postać algebraiczna

a – część rzeczywista $\quad b$ – część urojona

$i = \sqrt{-1}$ — jednostka urojona (w elektrotechnice $i \equiv j$)

$\underline{z} = ze^{j\delta}$ — postać wykładnicza

z – moduł ($z = |\underline{z}|$) $\quad \delta$ – argument

$\underline{z} = z(\cos\delta + j\sin\delta)$ — postać trygonometryczna

Przekształcanie postaci	
algebraiczna → wykładnicza	wykładnicza → algebraiczna
$z = \sqrt{a^2 + b^2} \qquad \delta = \begin{cases} \operatorname{arc\,tg}\frac{b}{a} & a > 0 \\ \operatorname{arc\,tg}\frac{b}{a} \pm \pi & a < 0 \end{cases}$	$a = z\cos\delta \qquad b = z\sin\delta$

Liczby zespolone

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Własności jednostki urojonej $\qquad j^2 = -1 \qquad \frac{1}{j} = \frac{j}{j^2} = -j \qquad j = 1e^{j90°}$

Podstawowe działania $\qquad \underline{z}_1 = a + jb = z_1 e^{j\delta} \qquad \underline{z}_2 = c + jd = z_2 e^{j\gamma}$

Dodawanie

$$\underline{z}_1 + \underline{z}_2 = (a + jb) + (c + jd) = (a + c) + j(b + d)$$

Mnożenie

$$\underline{z}_1\underline{z}_2 = (a + jb)(c + jd) = (ac - bd) + j(bc + ad)$$

$$\underline{z}_1\underline{z}_2 = z_1 e^{j\delta} z_2 e^{j\gamma} = z_1 z_2 e^{j(\delta+\gamma)}$$

Dzielenie

$$\frac{\underline{z}_1}{\underline{z}_2} = \frac{a + jb}{c + jd} = \frac{a + jb}{c + jd}\,\frac{c - jd}{c - jd} = \frac{(ac + bd) + j(bc - da)}{c^2 + d^2}$$

$$= \frac{ac + bd}{c^2 + d^2} + j\frac{bc - da}{c^2 + d^2}$$

$$\frac{\underline{z}_1}{\underline{z}_2} = \frac{z_1 e^{j\delta}}{z_2 e^{j\gamma}} = \frac{z_1}{z_2} e^{j(\delta-\gamma)}$$

Metoda amplitud zespolonych

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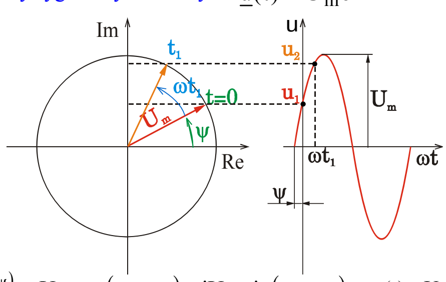
Zespolony sygnał wykładniczy $\qquad \underline{u}(t) = U_m e^{j(\omega t + \psi)}$



$$U_m e^{j(\omega t+\psi)} = U_m\cos(\omega t + \psi) + jU_m\sin(\omega t + \psi) \qquad u(t) = U_m\sin(\omega t + \psi)$$

$$\operatorname{Im}\{\underline{u}(t)\} = u(t)$$

Metoda amplitud zespolonych

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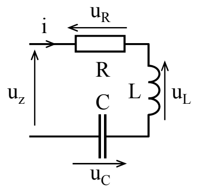

$u_z(t)=U_m\sin(\omega t+\psi)$

$\underline{u}_z(t)=U_m e^{j(\omega t+\psi)}=\boxed{U_m e^{j\psi}}e^{j\omega t}$

amplituda zespolona $\rightarrow \underline{U}_m$

$u_R+u_L+u_C=u_z$

$$Ri+L\frac{di}{dt}+\frac{1}{C}\int i\,dt=u_z \leftarrow$$ równanie dla sygnału sinusoidalnych

Podstawienie: $\underline{u}_z(t)=\underline{U}_m e^{j\omega t}$ $\quad \underline{i}(t)=\underline{I}_m e^{j\omega t}$

równanie dla zespolonych sygnałów wykładniczych $\rightarrow$ $$R\underline{i}(t)+L\frac{d\underline{i}(t)}{dt}+\frac{1}{C}\int \underline{i}(t)dt=\underline{u}_z(t)$$

Metoda amplitud zespolonych

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$$R\underline{I}_m e^{j\omega t}+L\frac{d\left(\underline{I}_m e^{j\omega t}\right)}{dt}+\frac{1}{C}\int \underline{I}_m e^{j\omega t}dt=\underline{U}_m e^{j\omega t}$$

$\boxed{\omega L=X_L}$ $\quad \boxed{\frac{1}{\omega C}=X_C,\ \frac{1}{j}=-j}$

$$R\underline{I}_m e^{j\omega t}+Lj\omega\underline{I}_m e^{j\omega t}+\frac{1}{Cj\omega}\underline{I}_m e^{j\omega t}=\underline{U}_m e^{j\omega t}\ \Big|\frac{1}{e^{j\omega t}}$$

równanie dla amplitud zespolonych $\rightarrow$ $R\underline{I}_m+jX_L\underline{I}_m-jX_C\underline{I}_m=\underline{U}_m\ \Big|\frac{1}{\sqrt{2}}$

równanie dla wartości skutecznych zespolonych $\rightarrow$ $R\underline{I}+jX_L\underline{I}-jX_C\underline{I}=\underline{U}_z$

Zespolone wartości skuteczne: $\underline{I}=Ie^{j\varphi}$ $\quad \underline{U}_z=U_z e^{j\psi}$

Metoda amplitud zespolonych

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$\underline{U}_z=\frac{U_m}{\sqrt{2}}e^{j\psi}$ – Fazor napięcia

Fazor (dawniej wskaz, wektor) – *ang.* phasor = **phase vector**

$$\underline{I}\left[R+j\left(X_L-X_C\right)\right]=\underline{U}_z$$

$$\underline{I}=\frac{\underline{U}_z}{R+j\left(X_L-X_C\right)}=Ie^{j\varphi}$$ – Fazor prądu

$$\underline{I}_m=\sqrt{2}Ie^{j\varphi}$$

$$\underline{i}(t)=\sqrt{2}Ie^{j\varphi}e^{j\omega t}=\sqrt{2}Ie^{j(\omega t+\varphi)}$$

$$i(t)=\mathrm{Im}\left\{\underline{i}(t)\right\}=\sqrt{2}I\sin(\omega t+\varphi)=I_m\sin(\omega t+\varphi)$$

Przykład liczbowy

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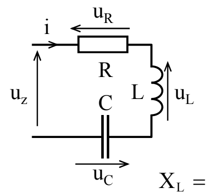
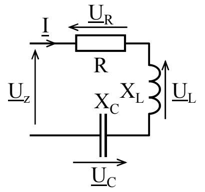


$u_z(t)=141{,}4\sin(400t+0)$ V

$R=10\,\Omega,\quad L=50\,\text{mH},\quad C=250\,\mu\text{F}$

$\underline{U}_z=\frac{141{,}4}{\sqrt{2}}e^{j0}\approx 100$ V

$X_L=400\cdot 0{,}05=20\,\Omega \quad X_C=\frac{1}{400\cdot 250\cdot 10^{-6}}=10\,\Omega$



$$\underline{I}=\frac{\underline{U}_z}{R+j\left(X_L-X_C\right)}=\frac{100}{10+j(20-10)}$$

$=(5-j5)$ A $=5\sqrt{2}e^{-j45°}\approx 7{,}07e^{-j45°}$ A

$\underline{i}(t)=\sqrt{2}5\sqrt{2}e^{-j45°}e^{j\omega t}=10e^{j(\omega t-45°)}$ A

$i(t)=10\sin(\omega t-45°)$ A $\quad I_{sk}=5\sqrt{2}\approx 7{,}07$ A

Przykład liczbowy

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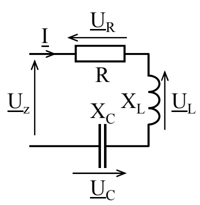


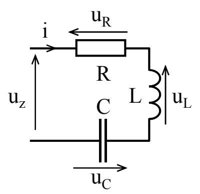
$\underline{U}_z=\boxed{R\underline{I}}+\boxed{jX_L\underline{I}}+\boxed{\left(-jX_C\right)\underline{I}}$

$\underline{U}_R \quad \underline{U}_L \quad \underline{U}_C$

$\underline{U}_R=R\underline{I}=10(5-j5)$
$=50-j50=70{,}7e^{-j45°}$ V

$\underline{U}_L=(jX_L)\underline{I}=j20(5-j5)$
$=100+j100=141{,}4e^{j45°}$ V

$\underline{U}_C=(-jX_C)\underline{I}=-j10(5-j5)$
$=-50-j50=70{,}7e^{-j135°}$ V



$u_R(t)=\boxed{70{,}7\sqrt{2}}\sin(\omega t-45°)$ V $\quad \boxed{U_{Rm}=100\text{ V}}$

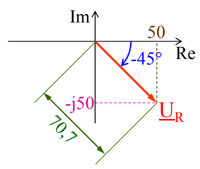
$u_L(t)=\boxed{141{,}4\sqrt{2}}\sin(\omega t+45°)$ V $\quad \boxed{U_{Lm}=200\text{ V}}$

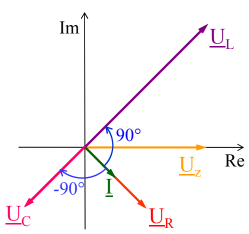
$u_R(t)=\boxed{70{,}7\sqrt{2}}\sin(\omega t-135°)$ V $\quad \boxed{U_{Cm}=100\text{ V}}$

Reprezentacja graficzna

24

$\underline{U}_R=50-j50=70{,}7e^{j(-45°)}$ V





$\underline{U}_z=\underline{U}_L+\underline{U}_R+\underline{U}_C$

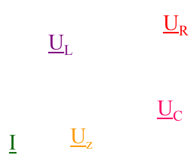
Wykres fazorowy (wskazowy)

$\underline{U}_z=100$ V

$\underline{U}_L=141{,}4e^{j45°}$ V

$\underline{U}_C=70{,}7e^{-j135°}$ V

$\underline{I}=7{,}07e^{-j45°}$ A



Oporność zespolona

$$\underline{I}=\frac{\underline{U}_z}{R+j\left(X_L-X_C\right)}$$

$$\frac{\underline{U}}{\underline{I}}=R+j\underbrace{\left(X_L-X_C\right)}_{X}=\underline{Z}$$ – impedancja zespolona (oporność pozorna)

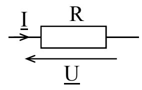
R – rezystancja [Ω] (oporność czynna) X – reaktancja [Ω] (oporność bierna)

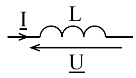
$$\frac{1}{\underline{Z}}=\underline{Y}$$ – admitancja zespolona (przewodność pozorna)

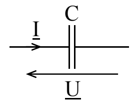
$$\underline{Y}=G+jB$$

G – konduktancja [Ω⁻¹=S] (przewodność czynna) B – susceptancja [Ω⁻¹=S] (przewodność bierna)

Impedancje i admitancje elementów







$$\underline{Z}_R=\frac{\underline{U}}{\underline{I}}=R \qquad \underline{Z}_L=\frac{\underline{U}}{\underline{I}}=j\omega L=jX_L \qquad \underline{Z}_C=\frac{\underline{U}}{\underline{I}}=\frac{1}{j\omega C}$$

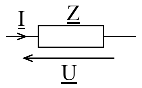
$$\underline{Y}_R=\underline{Z}_R^{-1}=\frac{1}{R}=G \qquad \underline{Z}_L=jX_L=X_Le^{j\frac{\pi}{2}} \qquad \underline{Z}_C=\frac{1}{j\omega C}=-j\frac{1}{\omega C}=-jX_C$$

$$\underline{Y}_L=\underline{Z}_L^{-1}=\frac{1}{j\omega L} \qquad \underline{Z}_C=-jX_C=X_Ce^{-j\frac{\pi}{2}}$$

$$\underline{Y}_L=\frac{1}{j\omega L}=-j\frac{1}{\omega L}=-jB_L \qquad \underline{Y}_C=\underline{Z}_C^{-1}=j\omega C=jB_C$$

$$\underline{Y}_L=-jB_L=B_Le^{-j\frac{\pi}{2}} \qquad \underline{Y}_C=jB_C=B_Ce^{j\frac{\pi}{2}}$$

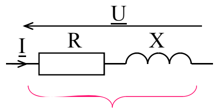
Impedancja – schemat dla ω=const.



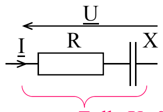
$$\underline{Z}=\frac{\underline{U}}{\underline{I}}$$

schemat szeregowy

$$\underline{Z}=R+jX$$



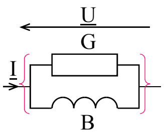
$\underline{Z}$ dla X>0



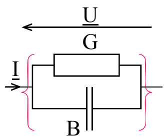
$\underline{Z}$ dla X<0

schemat równoległy

$$\underline{Y}=\underline{Z}^{-1}=G+jB$$

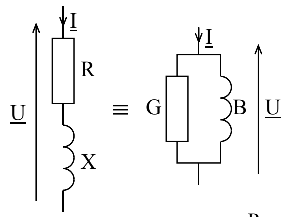


$\underline{Y}$ dla B<0



$\underline{Y}$ dla B>0

Impedancja – schemat dla ω=const.



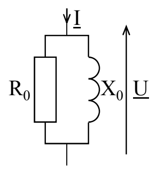
$$\underline{Y}=\underline{Z}^{-1}$$

$$\underline{Y}=G-jB$$

$$\underline{Z}^{-1}=\frac{1}{R+jX}=\frac{R-jX}{(R+jX)(R-jX)}$$

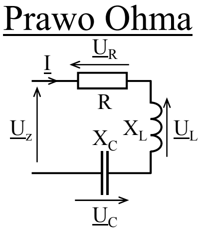
$$=\frac{R-jX}{R^2+X^2}=\frac{R}{R^2+X^2}-j\frac{X}{R^2+X^2}$$

$$G=\frac{R}{R^2+X^2} \qquad B=\frac{X}{R^2+X^2}$$



$$R_0=\frac{1}{G}=\frac{R^2+X^2}{R} \qquad X_0=\frac{1}{B}=\frac{R^2+X^2}{X}$$

Prawo Ohma

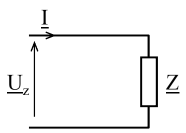


$$\underline{I}\underline{Z}_R+\underline{I}\underline{Z}_L+\underline{I}\underline{Z}_C=\underline{U}_z$$

$$\left(\underline{Z}_R+\underline{Z}_L+\underline{Z}_C\right)\underline{I}=\underline{U}_z$$

$$\underline{Z}=\underline{Z}_R+\underline{Z}_L+\underline{Z}_C$$

$$\underline{Z}\,\underline{I}=\underline{U}_z$$

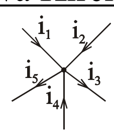


$$\underline{U}_z=\underline{Z}\,\underline{I}$$

Prawo Ohma w postaci zespolonej

Prawa Kirchhoffa w postaci zespolonej

I



$$i_1+i_2+i_4=i_3+i_5$$

$$\underline{i}_1+\underline{i}_2+\underline{i}_4=\underline{i}_3+\underline{i}_5$$

$$\underline{I}_{1m}e^{j\omega t}+\underline{I}_{2m}e^{j\omega t}+\underline{I}_{4m}e^{j\omega t}=\underline{I}_{3m}e^{j\omega t}+\underline{I}_{5m}e^{j\omega t}\ \Bigg|\ \frac{1}{\sqrt{2}e^{j\omega t}}$$

$$\underline{I}_1+\underline{I}_2+\underline{I}_4=\underline{I}_3+\underline{I}_5$$

$$\underline{I}_1+\underline{I}_2-\underline{I}_3+\underline{I}_4-\underline{I}_5=0$$

$$\boxed{\sum_{k=1}^{N}\underline{I}_k=0}$$

Prawa Kirchhoffa w postaci zespolonej

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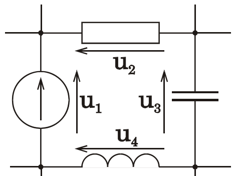
II



$$u_1 - u_2 - u_3 + u_4 = 0$$

$$\underline{u}_1 - \underline{u}_2 - \underline{u}_3 + \underline{u}_4 = 0$$

$$\underline{U}_{1m}e^{j\omega t} - \underline{U}_{2m}e^{j\omega t} - \underline{U}_{3m}e^{j\omega t} + \underline{U}_{4m}e^{j\omega t} = 0 \quad \Big| \frac{1}{\sqrt{2}e^{j\omega t}}$$

$$\underline{U}_1 - \underline{U}_2 - \underline{U}_3 + \underline{U}_4 = 0$$

$$\boxed{\sum_{k=1}^{N} \underline{U}_k = 0}$$

Impedancja zastępcza

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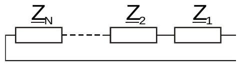
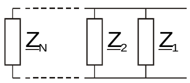
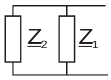


$$\underline{Z} = \sum_{k=1}^{N} \underline{Z}_k$$



$$\underline{Y} = \sum_{k=1}^{N} \underline{Y}_k \qquad \underline{Z} = \frac{1}{\underline{Y}}$$

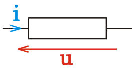
$$\underline{Y}_k = \frac{1}{\underline{Z}_k}$$



$$\underline{Z} = \frac{\underline{Z}_1 \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}$$

Moc w obwodzie prądu sinusoidalnego

33



$$u(t) = U_m \sin(\omega t + \varphi_u)$$

$$i(t) = I_m \sin(\omega t + \varphi_i)$$

Moc chwilowa $p(t) = u(t) \cdot i(t) = U_m I_m \sin(\omega t + \varphi_u)\sin(\omega t + \varphi_i)$

Moc czynna $P = \frac{1}{T}\int_0^T p\,dt = \frac{1}{2} U_m I_m \cos(\varphi_u - \varphi_i)$

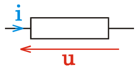
$$\varphi_u - \varphi_i = \varphi \qquad \frac{1}{2} U_m I_m = \frac{U_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = UI$$

Kąt przesunięcia fazowego między napięciem i prądem

$$\boxed{P = UI\cos\varphi}$$

Moc w obwodzie prądu sinusoidalnego

34

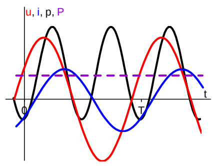


$$u(t) = U_m \sin(\omega t + \varphi_u)$$

$$i(t) = I_m \sin(\omega t + \varphi_i)$$

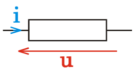
$$p = U_m I_m \sin(\omega t + \varphi_u)\sin(\omega t + \varphi_i)$$

$$P = UI\cos\varphi$$



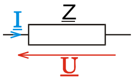
Moc w obwodzie prądu sinusoidalnego

35



$$u(t) = U_m \sin(\omega t + \varphi_u)$$

$$i(t) = I_m \sin(\omega t + \varphi_i)$$



$$\underline{U} = \frac{U_m}{\sqrt{2}} e^{j\varphi_u} = Ue^{j\varphi_u} \qquad \underline{I} = \frac{I_m}{\sqrt{2}} e^{j\varphi_i} = Ie^{j\varphi_i}$$

Moc pozorna zespolona $\underline{S} = \underline{U}\underline{I}^* \qquad \underline{I}^* = Ie^{-j\varphi_i}$

$$\underline{S} = UIe^{j(\varphi_u - \varphi_i)} = UI(\cos\varphi + j\sin\varphi) = UI\cos\varphi + jUI\sin\varphi$$

φ

P

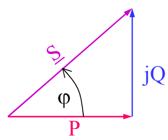
S - moc pozorna

Q - moc bierna

$$\underline{S} = P + jQ$$

Trójkąt mocy

36



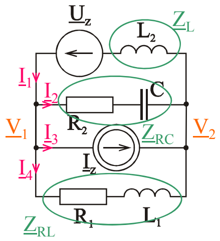
$$S = \sqrt{P^2 + Q^2}$$

$$Q = S\sin\varphi \qquad Q = P\,\mathrm{tg}\varphi$$

$$P = S\cos\varphi$$

$\cos\varphi$ – współczynnik mocy

Metoda potencjałów węzłowych



$$\underline{Z}_L = j\omega L_2 \qquad \underline{Z}_{RC} = R_2 - j\frac{1}{\omega C}$$

$$\underline{Z}_{RL} = R_1 + j\omega L_1$$

$$\underline{I}_1 - \underline{I}_2 - \underline{I}_3 - \underline{I}_4 = 0$$

$$\underline{I}_1 = \frac{\underline{U}_Z - \underline{V}_1 + \underline{V}_2}{\underline{Z}_L} \qquad \underline{I}_2 = \frac{\underline{V}_1 - \underline{V}_2}{\underline{Z}_{RC}}$$

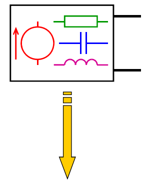
$$\underline{I}_3 = \underline{I}_Z \qquad \underline{I}_4 = \frac{\underline{V}_1 - \underline{V}_2}{\underline{Z}_{RL}}$$

$$\frac{\underline{U}_Z - \underline{V}_1 + \underline{V}_2}{\underline{Z}_L} - \frac{\underline{V}_1 - \underline{V}_2}{\underline{Z}_{RC}} - \underline{I}_Z - \frac{\underline{V}_1 - \underline{V}_2}{\underline{Z}_{RL}} = 0$$

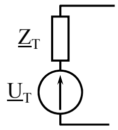
zał.: $\underline{V}_1$=0 $$\underline{V}_2 = \frac{\underline{I}_Z - \dfrac{\underline{U}_Z}{\underline{Z}_L}}{\dfrac{1}{\underline{Z}_L} + \dfrac{1}{\underline{Z}_{RC}} + \dfrac{1}{\underline{Z}_{RL}}} = \frac{\underline{I}_Z - \underline{U}_Z \underline{Y}_L}{\underline{Y}_L + \underline{Y}_{RC} + \underline{Y}_{RL}}$$

Twierdzenie Thevenina

liniowy układ aktywny

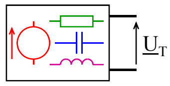


schemat zastępczy

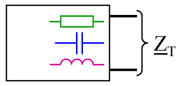


Określanie parametrów $\underline{U}_T$, $\underline{Z}_T$

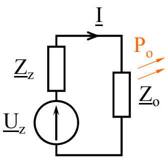
dwójnik aktywny



dwójnik pasywny



Dopasowanie mocowe



$$\underline{Z}_o = ? \Rightarrow P_o = P_{max}$$

$$\underline{Z}_z = R_z + jX_z \qquad \underline{Z}_o = R_o + jX_o$$

$$\underline{I} = \frac{\underline{U}_z}{\underline{Z}_z + \underline{Z}_o} = \frac{\underline{U}_z}{R_z + R_o + j(X_z + X_o)}$$

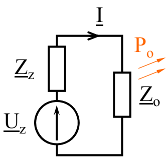
$$P_o = I^2 R_o \qquad I = \frac{U_z}{\sqrt{(R_z+R_o)^2 + (X_z+X_o)^2}}$$

$$P_o = \frac{U_z^2}{(R_z+R_o)^2 + (X_z+X_o)^2} R_o$$

$$\frac{\partial P_o}{\partial X_o} = U_z^2 R_o \frac{2(X_z+X_o)}{\left[(R_z+R_o)^2 + (X_z+X_o)^2\right]^2} = 0 \ \Leftrightarrow\ X_z + X_o = 0$$

$$\boxed{X_o = -X_z}$$

Dopasowanie mocowe



$$P_o = \frac{U_z^2}{(R_z+R_o)^2 + (X_z+X_o)^2} R_o$$

$$\frac{\partial P_o}{\partial R_o} = U_z^2 \frac{(R_z+R_o)^2 + \underbrace{(X_z+X_o)}_{0}{}^2 - 2R_o(R_z+R_o)}{\left[(R_z+R_o)^2 + \underbrace{(X_z+X_o)}_{0}{}^2\right]^2}$$

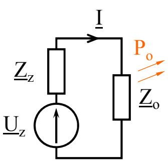
$$= U_z^2 \frac{R_z^2 - R_o^2}{(R_z+R_o)^4} = 0 \ \Leftrightarrow\ R_z^2 - R_o^2 = 0$$

$$\boxed{R_o = R_z} \qquad \cancel{R_o = -R_z}$$

$$\underline{Z}_o = R_z - jX_z = \underline{Z}_z^*$$

$$P_o = \frac{U_z^2}{(R_z+R_z)^2 + (X_z-X_z)^2} R_z = \frac{U_z^2}{(2R_z)^2} R_z = \frac{U_z^2}{4R_z}$$

Dopasowanie mocowe



$$\underline{Z}_z = R_z + jX_z$$

$$\underline{Z}_o = R_z - jX_z$$

moc źródła $\underline{S}_z = \underline{U}_z \underline{I}^*$

$$\underline{I} = \frac{\underline{U}_z}{R_z + R_z + j(X_z - X_z)} = \frac{\underline{U}_z}{2R_z}$$

$$\underline{I}^* = \frac{\underline{U}_z^*}{2R_z}$$

$$\underline{S}_z = \frac{\underline{U}_z \underline{U}_z^*}{2R_z} = \frac{U_z^2}{2R_z} = 2P_o = P_z$$

sprawność $\eta = \frac{P_o}{P_z} 100\% = 50\%$