



**Fundusze Europejskie**  
Wiedza Edukacja Rozwój

**Unia Europejska**  
Europejski Fundusz Społeczny



### **The Conley index for flows.**

### **The homotopy $LS$ -index and strongly indefinite problems.**

### **Applications of the $LS$ -index to certain ODE's and PDE's of variational type.**

Forma: odczyt, Liczba godz. 3

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An outline of lectures:

A brief introduction to the Conley index theory for flows in a locally compact spaces will be given in the first lecture. It is not our intention to develop the theory rigorously but the aim is to provide an intuitive understanding of the index. A short presentation of preliminary definitions and basic facts in the theory of Conley will be given. Next, a few examples will be discussed to illustrate the most important properties of the index.

Lecture two will be devoted to an extension of the classical Conley index to flows on an infinite-dimensional Hilbert space  $H$  generated by vector fields  $f : H \rightarrow H$ ,  $f(x) = Lx + K(x)$ , Where  $L : H \rightarrow H$  is a bounded linear operator satisfying certain technical assumptions and  $K$  is a completely continuous perturbation. An example will be discussed to show how this new invariant, called the  $LS$ -index, can be applied in searching critical points of strongly indefinite functionals having asymptotically linear gradient.

Multiplicity problems for periodic solutions of certain types of Hamiltonian systems and for solutions of indefinite elliptic systems will be considered in lecture three. The  $LS$ -index will be used as a tool.

References:

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