The Spectral Flow in Variational Bifurcation theory
Forma: odczyt, Liczba godz. 3

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Abstract
A special feature in nonlinear analysis is the existence of multiple solutions, and bifurcation is a paradigm for this non-uniqueness. The basic idea of bifurcation theory is to study equations $F(\lambda,u) = 0$, where $F : R\times X \rightarrow X$ is a sufficiently smooth family of maps on some Banach space X such that $F(\lambda,0) = 0$ for all $\lambda \in R$. A bifurcation point is a parameter value λ_* at which non-trivial solutions branch off from the trivial ones $R\times\{0\}$. There are numerous applications where this setting appears in science and engineering, e.g., the buckling of a rod in statics, the appearance of Taylor vortices in fluid dynamics and the onset of oscillations in an electric circuit in electrical engineering.
In the first part of this lecture series, we recap the definition of a bifurcation point, discuss some examples and explain that the derivative $L_{\lambda_*} := D_0F(\lambda_*,\cdot) \in L(X)$ is not invertible if λ_* is a bifurcation point. Afterwards, we briefly review the classical Krasnoselski bifurcation theorem for maps of the type $F(\lambda,u) = \lambda u - G(u)$, where $D_0G : X\rightarrow X$ is assumed to be compact.
In the second part of the series, we consider Hilbert spaces and assume that $L_\lambda = \lambda id_H - D_0G$ is selfadjoint. We begin by discussing the well known principle that a jump in the Morse index of L_λ, when λ travels along the real line, implies the existence of a bifurcation point. Afterwards, we introduce the spectral flow for paths of (bounded) selfadjoint Fredholm operators, which is an integer alued homotopy invariant that was introduced by Atiyah, Patodi and Singer in the seventies. We explain a bifurcation theorem of Fitzpatrick, Pejsachowicz and Recht from [1] (in an improved version of Pejsachowicz (Politecnico di Torino) and myself from [2]), which treats more general equations $F(\lambda,u) = 0$ than the ones in Krasnoselski's theorem and shows that the non-vanishing of the spectral flow of the path $L_\lambda := D_0F(\lambda,\cdot)$, $\lambda\in R$, entails the existence of a bifurcation point. Finally, we show thatKrasnoselski's theorem in Hilbert spaces is a corollary of this result.
In the final part of this lecture series, we introduce a bifurcation theorem for non-cooperative elliptic systems from [4], which is based on a comparison principle for the spectral flow from [2] and which generalises a Theorem by Szulkin from [3].

References
[1] P.M. Fitzpatrick, J. Pejsachowicz, L. Recht, **Spectral Flow and Bifurcation of Critical Points of Strongly-Indefinite Functionals Part I: General Theory**, J. Funct. Anal. 162, 1999, 52–95

[2] J. Pejsachowicz, N. Waterstraat, **Bifurcation of critical points for continuous families of C2 functionals of Fredholm type**, J. Fixed Point Theory Appl. 13, 2013, 537–560, arXiv:1307.1043 [math.FA]

[3] A. Szulkin, **Bifurcation for strongly indefinite functionals and a Liapunov type theorem for Hamiltonian systems**, J. Differential Integral Equations 7, 1994, 217–234

[4] N. Waterstraat, **Spectral flow and bifurcation for a class of strongly indefinite elliptic systems**, to appear in Proc. Royal Soc. Edinburgh A, arXiv:1512.04109 [math.AP]