Exercise 1. Check the continuity of the following functions and sketch their graphs if they reduce to elementary functions. Determine the types of the points of discontinuity (if they exist).

$a(x) = \frac{4-x^2}{|4x-x^3|}$, $b(x) = \frac{\sin x}{x}$, $c(x) = \frac{\arcsin(x+2)}{x^2+2x}$,

$$d(x) = \begin{cases} 2^x + 3 & x \leq 0 \\ (x-2)^2 & x > 0 \end{cases}, \quad e(x) = \begin{cases} 1 - x - x^2 & x \leq 0 \\ 1 + \log(x+1) & x > 0 \end{cases}, \quad f(x) = \begin{cases} 2^x & -1 \leq x \leq 0 \\ -x+1 & 0 < x \leq 1 \\ \log x & 1 < x \leq 2 \end{cases},$$

$$g(x) = \begin{cases} \frac{\sin^2 x}{x\sqrt{x^2}} & x \neq 0 \\ -1 & x = 0 \end{cases}, \quad h(x) = \begin{cases} 1 + \arctan x & x \leq 0 \\ \ln x & x > 0 \end{cases}, \quad i(x) = \begin{cases} \frac{\pi}{2}x & x < -1 \\ \arcsin x & -1 \leq x \leq 1 \\ \ln x + \frac{\pi}{2} & x > 1 \end{cases}.$$

Exercise 2. Determine the value of parameter α (and in one case: β) for which the following functions are continuous for every $x \in \mathbf{R}$.

$$a(x) = \begin{cases} 4 - (x+1)^2 & x < 2 \\ x + \alpha & x \geq 2 \end{cases}, \quad b(x) = \begin{cases} \frac{x^3-1}{1-x} & x \neq 1 \\ 6\alpha^2 - \alpha - 5 & x = 1 \end{cases},$$

$$c(x) = \begin{cases} e^{\frac{\sin x}{|x|}} & x \neq 0 \\ \alpha & x = 0 \end{cases}, \quad d(x) = \begin{cases} 2\alpha x + 6 & x \leq 1 \\ \frac{x^2-1}{1-x} & 1 < x < 2 \\ \beta x^2 - 1 & x \geq 2 \end{cases}.$$

Exercise 3. Give your own example of a function (i.e. give a formula and sketch the graph) that satisfies the following conditions:

a) has three points of discontinuity (one of the first type and two other one of the second type) and is increasing,

b) has three points of discontinuity and is even,

c) has infinitely many points of discontinuity.

Most exercises were taken from the script ”Matematyka - podstawy z elementami matematyki wyższej” issued by the Gdańsk University of Technology publishing house.