Part I - Integration Techniques (*theory file: BC_t_integrals_1.pdf*)

Exercise 1. Evaluate the following integrals without resorting to integration by parts and integration by substitution – you may use only basic formulas. If necessary, refer to Examples 1 – 7.

a) $\int 2^{2x}e^x dx$, b) $\int \tan^2 x dx$, c) $\int (\tan x + \cot x)^2 dx$, d) $\int \cot^2 x dx$,
e) $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$, f) $\int 3^x 2^x dx$, g) $\int \frac{4-x}{2+\sqrt{x}} dx$, h) $\int \frac{dx}{1-x^2}$,
i) $\int \frac{dx}{\sin^2 x \cos^2 x}$, j) $\int \frac{x^2-1}{x^2+1} dx$, k) $\int \frac{x+1}{\sqrt{x}} dx$, l) $\int \sin^2 \frac{x}{2} dx$.

Exercise 2. Evaluate the following integrals by parts. If necessary, refer to Examples 8 and 9.

a) $\int \sin^2 x dx$, b) $\int e^x \cos x dx$, c) $\int \ln^2 x dx$, d) $\int \arctan x dx$,
e) $\int \log_2 x dx$, f) $\int x \cos x dx$, g) $\int e^{2x} \cos(3x) dx$, h) $\int x^2 \sin x dx$,
i) $\int \frac{x}{\sin^2 x} dx$, j) $\int x \ln x dx$, k) $\int x \ln(1+\frac{1}{x}) dx$, l) $\int x \arctan x dx$.

Exercise 3. Evaluate the following integrals by substitution. If necessary, refer to Examples 10 – 12.

a) $\int \sqrt[3]{4x+3} dx$, b) $\int \frac{e^{2x} dx}{e^x+1}$, c) $\int x e^{x^2} dx$, d) $\int x\sqrt{x^2+9} dx$,
e) $\int \frac{dx}{x \ln^2 x}$, f) $\int \frac{\cos(\ln x) dx}{x}$, g) $\int \frac{1}{x \ln x} dx$, h) $\int \frac{x}{1+x^4} dx$,
i) $\int \frac{1}{e^x+e^{-x}} dx$, j) $\int \sin^3 x \cos x dx$, k) $\int \frac{1}{\sqrt{x}+1} dx$, l) $\int \frac{x}{\sqrt{2-4\ln^2 x}} dx$,
m) $\int \frac{\sqrt{x+1}}{\sqrt{x+1}+1} dx$, n) $\int \frac{2x+\sqrt{x-1}}{\sqrt{x-1}} dx$, o) $\int \frac{\sqrt{x}}{x+1} dx$, p) $\int \frac{\sin x}{\sqrt{1+2\cos x}} dx$.

Exercise 4. Evaluate the following integrals. If necessary, refer to Examples 13 – 16.

a) $\int \frac{1}{x^2-3x+1} dx$, b) $\int \frac{1}{x^2+4x+5} dx$, c) $\int \frac{1}{x^2+5x+6} dx$, d) $\int \frac{2x-3}{x^3+xdx}$,
e) $\int \frac{2}{(x-3)(x+5)} dx$, f) $\int \frac{5x}{(x-1)(x-2)(x-3)} dx$, g) $\int \frac{x-1}{(x+1)(x+2)} dx$, h) $\int \frac{x}{x^2-5x+6} dx$,
i) $\int \frac{3x^2-2x+1}{(x-1)(x-2)(x-3)} dx$, j) $\int \frac{1}{x^4-1} dx$, k) $\int \frac{2x+1}{3x^2-2x-1} dx$, l) $\int \frac{x^5-2}{x^2-1} dx$.

Exercise 5. Evaluate the following integrals. If necessary, refer to Example 17.

a) $\int \sqrt{x^2-9} dx$, b) $\int \sqrt{x^2+4} dx$, c) $\int \sqrt{x^2+9} dx$.

Exercise 6. Evaluate the following integrals. If necessary, refer to Examples 18 and 19.

a) $\int \sin(5x+\pi)\cos(3x-\pi) dx$, b) $\int \sin(3x)\sin(3x+\frac{\pi}{2}) dx$, c) $\int \cos(6x-\pi)\cos(3x+\frac{\pi}{2}) dx$,
d) $\int \frac{2dx}{2+\cos x}$, e) $\int \frac{dx}{1+\sin x}$, f) $\int \frac{dx}{\tan \frac{x}{2}}$.

Part II - Applications (*theory file: BC_t_integrals_2.pdf*)

Exercise 7. Calculate the following definite integrals.

a) $\int_1^3 \sqrt{1+x} dx$, b) $\int_{12}^{13} \frac{2x}{4x^2+1} dx$, c) $\int_0^e \ln(x+1) dx$, d) $\int_0^{\sqrt{3}} x \arctan(x^2) dx$,
e) $\int_0^1 shx dx$, f) $\int_{\frac{1}{e}}^0 \ln x dx$, g) $\int_0^{\frac{1}{2}} \frac{dx}{x^2+x+1}$, h) $\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos^2 x dx$.

Exercise 8. Calculate the following areas. Sketch a diagram in each case.

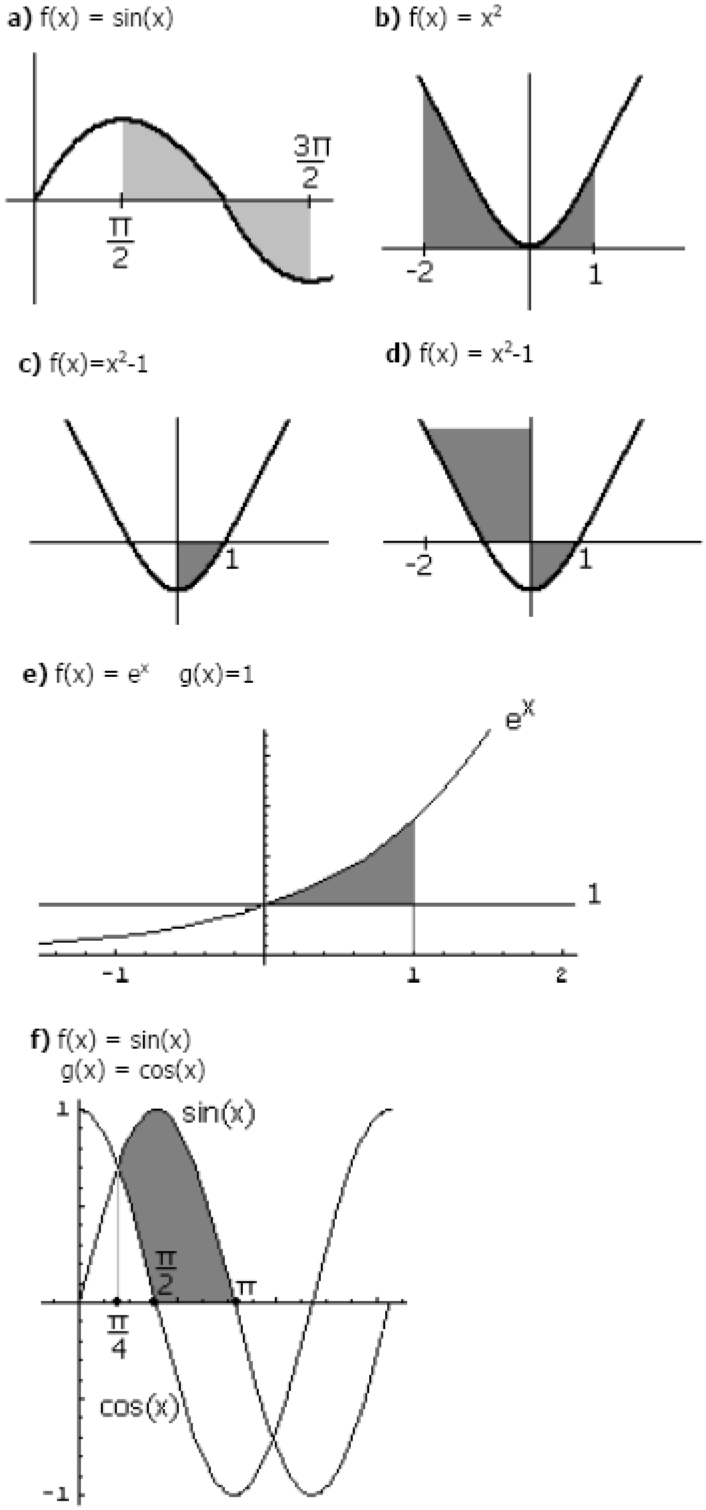
a) Calculate the area bounded by the graph of a function e^x and lines $x=0$ and $x=1$.

b) Calculate the area bounded by graphs of functions $g(x)=x^2$ and $f(x)=\sqrt{x}$.

c) Calculate the area bounded by graphs of functions $g(x)=e$ and $f(x)=e^x$ and by a line $x=0$.

d) Calculate the area bounded by graphs of functions $g(x)=\sin x$ and $f(x)=1$ and a line $x=0$.

Exercise 9. Calculate the shaded areas.



Exercise 10. Draw a solid of a revolution of function $f(x) = -x^2 + 1$ in the interval of $[-2, 2]$. Calculate the volume of the solid.

Exercise 11. Draw a solid of a revolution of function $f(x) = \tan x$ in the interval of $[0, \frac{\pi}{4}]$. Calculate the volume of the solid.

Exercise 12. Function f is given by the following formula:

$$f(x) = \begin{cases} \frac{1}{e} & x < -1 \\ e^x & -1 \leq x \leq 1 \\ e & x > 1 \end{cases}$$

Draw the diagram of function $f(x)$ and then draw a solid of a revolution of $f(x)$ in the interval of $[-2, 5]$. Calculate the volume of the solid.

Exercise 13. Let R be a rectangle bounded by lines $x = -1,\ x = 1$ and by graphs of $f(x) = 4$ and $g(x) = 4.5$. Calculate the volume of a solid of revolution obtained by revolving rectangle R about the OX axis. Can you calculate this volume without using integrals?

Exercise 14. Let T be a triangle bounded by graphs of functions $f(x) = \frac{x}{2} + 4$ and $g(x) = x + 4$ and by the line $x = 1$. Calculate the volume of a solid of revolution obtained by revolving triangle T about the OX axis.

Exercise 15. Draw a solid of a revolution of a function $f(x) = x + 1$ in the interval of $[0, 5]$ and then calculate its surface. What is the complete surface of this solid?

Exercise 16. Draw a solid of a revolution of a function $f(x) = 2x + 1$ in the interval of $[1, 3]$ and then calculate its complete surface.

Exercise 17. Calculate the lengths of curves given in Exercises 15 and 16.

Exercise 18. Calculate the length of curve $f(x) = x^2$ for $x \in [-1, 1]$. Hint: use hyperbolic functions.