Exercise 1. Calculate the following limits using various transformations of function formulas. For details refer to Examples 1, 2, 3, 6, 7 and 8 from the theory PDF file.

a) $\lim\limits_{x\to 2}\frac{x+1}{x-3}$, b) $\lim\limits_{x\to -1}\frac{x^2+2x+1}{x^2+x}$, c) $\lim\limits_{x\to 4}\frac{x-4}{2-\sqrt{x}}$, d) $\lim\limits_{x\to 0}\frac{\sqrt{x+1}-1}{x}$,
e) $\lim\limits_{x\to 1}\frac{x-1}{1-\sqrt{2-x}}$, f) $\lim\limits_{x\to 0}\frac{x+1-\sqrt{1-2x-x^2}}{2x}$, g) $\lim\limits_{x\to 5}\frac{x^2-25}{\sqrt{x-4}-1}$, h) $\lim\limits_{x\to 0}\frac{2-\sqrt[3]{x+8}}{x}$,
i) $\lim\limits_{x\to\infty}(x+1-\frac{1}{x+1})$, j) $\lim\limits_{x\to\infty}\frac{4-9x}{1+3x}$, k) $\lim\limits_{x\to\infty}\frac{\sqrt{9x^2+2x-6}}{2-3x}$, l) $\lim\limits_{x\to-\infty}\frac{\sqrt{x^2+1}}{-x}$,
m) $\lim\limits_{x\to\infty}(\sqrt{x^2+1}-x)$, n) $\lim\limits_{x\to\infty}\frac{\sqrt{2x+1}-\sqrt{3x+1}}{x}$, o) $\lim\limits_{x\to\infty}\frac{1}{\sqrt{x}(\sqrt{x}-\sqrt{x-1})}$, p) $\lim\limits_{x\to\infty}\frac{10^{\sqrt{x^2+2x+3}}}{10^{\sqrt{x^2+1}}}$.

Exercise 2. Calculate the following limits using property $\lim\limits_{x\to 0}\frac{\sin x}{x}=1$. For details refer to Examples 4 and 5.

a) $\lim\limits_{x\to 0}\frac{\sin 8x}{4x}$, b) $\lim\limits_{x\to 0}\frac{\sin 7x}{\sin 64x}$, c) $\lim\limits_{x\to 0}\frac{x}{\tan 8x}$, d) $\lim\limits_{x\to 0}4x\cot 7x$,
e) $\lim\limits_{x\to 0}\frac{2x}{\sin 4x-\sin 5x}$, f) $\lim\limits_{x\to 0}\frac{6x+\sin 2x}{\sin 3x-5x}$, g) $\lim\limits_{x\to 1}\frac{\sin(x-1)}{1-x^2}$, h) $\lim\limits_{x\to\frac{\pi}{3}}\frac{\sin(x-\frac{\pi}{3})}{1-2\cos x}$.

Exercise 3. Calculate the following limits, refer to Example 10 for details.

a) $\lim\limits_{x\to\infty}\frac{3^{x+2}+4^x}{6^x-3^x}$, b) $\lim\limits_{x\to\infty}\frac{2^{x+1}-3^{x+2}}{3^{x-1}}$, c) $\lim\limits_{x\to\infty}\frac{e^x+e^{-x}}{e^x-e^{-x}}$, d) $\lim\limits_{x\to-\infty}\frac{\pi^{2x}+\pi^{-2x}}{\pi^{2x}-\pi^{-2x}}$.

Exercise 4. Calculate the following limits using property $\lim\limits_{x\to\pm\infty}(1+\frac{1}{x})^x=e$. For details refer to Examples 11 and 12.

a) $\lim\limits_{x\to\infty}(\frac{x+1}{x-2})^{2x-1}$, b) $\lim\limits_{x\to\infty}(\frac{2x+1}{x-1})^x$, c) $\lim\limits_{x\to\infty}(\frac{x^2+2}{x^2-3})^{\frac{x^2}{5}}$,
d) $\lim\limits_{x\to\infty}(\frac{7x+3}{7x-5})^{x^2}$, e) $\lim\limits_{x\to\infty}(\frac{x^2-2}{x^2+1})^{\frac{x}{3}}$, f) $\lim\limits_{x\to\infty}(\frac{3x+2}{3x-2})^{2x}$.

Exercise 5. Calculate the following limits, for details refer to Example 9.

a) $\lim\limits_{x\to\infty}\frac{\sin x}{x}$, b) $\lim\limits_{x\to\infty}\frac{5x}{7x^2+4}\cos x$, c) $\lim\limits_{x\to\infty}\frac{\cos^2 x}{-x^2-4}$, d) $\lim\limits_{x\to-\infty}\frac{2x\sin x}{7x-x^7}$.

Exercise 6. Calculate limits of the following function compositions. For details refer to Examples 13 and 14.

a) $\lim\limits_{x\to\infty}\sin(\arctan x)$, b) $\lim\limits_{x\to\infty}\arcsin(\frac{1-x}{1+x})$, c) $\lim\limits_{x\to\infty}\arccos(\frac{2+x}{2-x})$, d) $\lim\limits_{x\to\infty}\log_2|\frac{x+1}{x^2+2}|$.

Exercise 7. Calculate right-hand and left-hand limits of the following functions in given points. For details see Examples 15, 16 and 17.

$a(x)=\frac{x^2-5}{x}$, $x_0=0$, $b(x)=\frac{4x-3}{(5-x)^2}$, $x_0=5$, $c(x)=\frac{x^2-2}{x^2-2x-3}$, $x_0=-1$,
$d(x)=2^{\frac{1}{(1-x)^2}}$, $x_0=1$, $e(x)=\frac{5^{\frac{1}{x}}}{1+5^{\frac{1}{x}}}$, $x_0=0$, $f(x)=e^{\frac{\pi}{1-x}}$, $x_0=1$,
$g(x)=\frac{2^{\frac{1}{x}}+6}{6^{\frac{1}{x}}+2}$, $x_0=0$, $h(x)=\arctan(\frac{\sin x}{|x|})$, $x_0=0^-$, $i(x)=\log(\frac{2}{x^2})$, $x_0=0^+$.

Exercise 8. Check if limits of the following functions exist in given points. If so, calculate their values. For details see Example 18.

$a(x)=\frac{1-x^2}{|x+1|}$, $x_0=-1$, $b(x)=\frac{|x^2-1|}{\sqrt{2-x}-2}$, $x_0=1$, $c(x)=\frac{\tan(2|x|)}{3x}$, $x_0=0$,

$$d(x)=\begin{cases}\frac{x+1}{x-2} & x<-1\\ x^2-1 & x>-1\end{cases},\ x_0=-1, \quad e(x)=\begin{cases}x^3-3 & x<1\\ \frac{1-x}{x+4} & x>1\end{cases},\ x_0=1.$$

Most exercises were taken from the script "Matematyka - podstawy z elementami matematyki wyższej" issued by the Gdańsk University of Technology publishing house.