Exercise 1. Express the following values in radians: $9°$, $15°$, $75°$, $30°$, $45°$, $60°$, $90°$, $180°$.

Exercise 2. Express the following values in degrees: $\frac{5\pi}{6}$, $\frac{-9\pi}{4}$, $\frac{\pi}{15}$, $\frac{-\pi}{18}$, $\frac{\pi}{360}$.

Exercise 3. Calculate exact values of the following expressions.

   a) $\cos\left(-1050°\right) + \sin 870°$,   b) $9\sin 120° \cdot \tan 300°$,   c) $\sin 75° + \sin 15°$,   d) $\frac{\cos 300°}{\cos 30°}$,   e) $\sin \frac{17\pi}{6}$.

Trigonometric identities

$$\sin^2 \alpha + \cos^2 \alpha = 1 \qquad\qquad \tan \alpha \cdot \cot \alpha = 1 \text{ for } \alpha \neq k \cdot \frac{\pi}{2},\ k \in \mathbf{Z}$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} \text{ for } \alpha \neq \frac{\pi}{2} + k\pi,\ k \in \mathbf{Z} \qquad\qquad \cot \alpha = \frac{\cos \alpha}{\sin \alpha} \text{ for } \alpha \neq k\pi,\ k \in \mathbf{Z}$$

$$\sin\left(\alpha \pm \beta\right) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \qquad\qquad \cos\left(\alpha \pm \beta\right) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan\left(\alpha + \beta\right) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \qquad\qquad \text{for } \cos\left(\alpha + \beta\right) \neq 0 \text{ and } \cos \alpha \cdot \cos \beta \neq 0$$

$$\cot\left(\alpha + \beta\right) = \frac{\cot \alpha \cot \beta - 1}{\cot \alpha + \cot \beta} \qquad\qquad \text{for } \sin\left(\alpha + \beta\right) \neq 0 \text{ and } \sin \alpha \cdot \sin \beta \neq 0$$

Exercise 4. Using formulas from the above table construct formulas for:

   a) $\sin\left(2\alpha\right)$,   b) $\cos\left(2\alpha\right)$ - construct three different formulas,   c) $\tan\left(2\alpha\right)$,

   d) $\sin\left(\frac{\pi}{2} - x\right)$,   e) $\cos\left(\frac{\pi}{2} - x\right)$,   f) $\sin\left(\frac{\pi}{2} + x\right)$.

Exercise 5. Give an example of angles x and y showing that:

   a) $\sin x + \sin y \neq \sin\left(x + y\right)$,   b) $\sin x \cdot \sin y \neq \sin(x \cdot y)$,

   c) $\cos x + \cos y \neq \cos\left(x + y\right)$,   d) $\cos x \cdot \cos y \neq \cos(x \cdot y)$.

Exercise 6. Solve the following equations.

   a) $\sin x = \frac{\sqrt{2}}{2}$,   b) $\cos x = -\frac{3}{2}$,   c) $\tan x = 1$,

   d) $\cot x = -\sqrt{3}$,   e) $\sin x + \sin 3x = \sin 2x$,   f) $6\sin^2 x + 7\cos x - 1 = 0$,

   g) $\tan x = \sin x$,   h) $2\sin x = 3\cot x$,   i) $\cos^2 x = \frac{3}{4}$,

   j) $4\sin^3 x + 2 = 1 - 2\sin^2 x - 2\sin x$,   k) $\sin x + \cos x = 0$,   l) $\tan x + \cot x = \frac{4\sqrt{3}}{3}$.

Exercise 7. Solve the following inequalities.

   a) $\sin x > \frac{1}{2}$,   b) $\sin x \geq \frac{1}{2}$,   c) $\sin x \leq \frac{1}{2}$,

   d) $\sin x < \frac{1}{2}$,   e) $\cos x \leq 1 - 2\cos^2 x$,   f) $\cot\left(2x - \frac{\pi}{4}\right) \geq -1$,

   g) $\cos x + \tan x \leq 1 + \sin x$,   h) $\sin 2x > \sin x$,   i) $\tan 3x \geq 1$.

Exercise 8. Set the domain of the following functions.

   a) $f(x) = \sqrt{1 - \sin^2 x}$,   b) $f(x) = \sqrt{1 - x^2} - 2\log\left(\sin x\right)$,   c) $f(x) = \log\left(\cos\left(\log x\right)\right)$.

Exercise 9. Calculate the exact value of the following expressions.

   a) $\arccos \frac{\sqrt{2}}{2} + \arctan\left(-1\right)$,   b) $\tan\left(\arcsin \frac{\sqrt{2}}{2}\right)$,   c) $\frac{-\arctan \sqrt{3} + 3\arctan \frac{\sqrt{3}}{3}}{\arccos 0}$.

Exercise 10. Solve equations: (a) $4(\arcsin x)^2 - \pi^2 = 0$,   (b) $\cos(\arcsin(2x)) = \frac{1}{2}$.

Exercise 11. Sketch graphs of the following functions, set their domain and range.

   a) $f(x) = \pi - \arctan(1 - x)$,   b) $g(x) = 2\arcsin \frac{3x+2}{5} + \frac{\pi}{2}$.