Inverse property of indefinite integrals

$$\left[\int f(x)dx\right]' = f(x)$$
$$\int f'(x)dx = f(x) + C$$

Antiderivative formulas

$$\int 0\, dx = C$$
$$\int x^p\, dx = \frac{x^{p+1}}{p+1} + C, \quad p \neq -1$$
$$\int \frac{1}{x}\, dx = \ln|x| + C, \quad x \neq 0$$
$$\int a^x\, dx = \frac{a^x}{\ln a} + C\,, \qquad \int e^x\, dx = e^x + C$$
$$\int \sin x\, dx = -\cos x + C\,, \qquad \int \cos x\, dx = \sin x + C$$
$$\int \frac{1}{\cos^2 x}\, dx = \tan x + C, \quad x \neq \frac{\pi}{2} + k\pi$$
$$\int \frac{1}{\sin^2 x}\, dx = -\cot x + C, \quad x \neq k\pi$$
$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \arcsin x + C = -\arccos x + C, \quad x \in (-1, 1)$$
$$\int \frac{1}{1+x^2}\, dx = \arctan x + C = -\text{arccot}\ x + C$$

Antiderivative linearity rules

$$\text{Sum Rule:} \int (f(x) + g(x))\, dx = \int f(x)\, dx + \int g(x)\, dx$$
$$\text{Difference Rule:} \int (f(x) - g(x))\, dx = \int f(x)\, dx - \int g(x)\, dx$$
$$\text{Constatnt Multiple Rule:} \int (a \cdot f(x))\, dx = a \cdot \int f(x)\, dx\ , \text{ where } a \in \mathbf{R}.$$

Antiderivative formulas for functions and their derivatives

$$\int (f(x))^n \cdot f'(x) dx = \tfrac{(f(x))^{n+1}}{n+1} + C$$
$$\int \tfrac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$
$$\int \tfrac{f'(x)}{(f(x))^2} dx = \tfrac{1}{f(x)} + C$$
$$\int e^{f(x)} \cdot f'(x) = e^{f(x)} + C$$
$$\int \tfrac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + C$$

Integration by Parts

If u and v have continuous derivatives, then

$$\int f(x)g'(x)\, dx = f(x)g(x) - \int f'(x)g(x)\, dx.$$

Integration by substitution

If $u = g(x)$ is a diferentiable function whose range is an interval A and f is continuous in A, then

$$\int f(x)\,dx = \int f(g(t))g\,'(t)\,dt = F(g(t)) + C\,.$$

Integrals of the form: $\int \frac{dx}{ax^2+bx+c}$, $\Delta < 0$

$$\int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{4ac-b^2}} \cdot \arctan(\frac{2ax+b}{\sqrt{4ac-b^2}})$$

Integration by partial fractions

$$\int \frac{Adx}{(x+a)} = A \cdot \ln|x+a| + C, \qquad A, C \in \mathbf{R}$$

$$\int \frac{Adx}{(x+a)^n} = \frac{-A}{(n-1)(x+a)^{n-1}} + C, \qquad A, C \in \mathbf{R},\ n \geq 2$$

$$\int \frac{dx}{(1+x^2)^n} = \frac{x}{2(n-2)(1+x^2)^{n-1}} + \frac{2n-3}{2(n-1)} \int \frac{dx}{(1+x^2)^{n-1}}, \qquad n \geq 2$$

Hyperbolic Functions and their properties

$$\mathrm{sh}(x) = \frac{e^x-e^{-x}}{2} \qquad \mathrm{ch}(x) = \frac{e^x+e^{-x}}{2}$$

$$\mathrm{sh}'(x) = \mathrm{ch}(x)$$

$$\mathrm{ch}'(x) = \mathrm{sh}(x)$$

$$1 = \mathrm{ch}^2(x) - \mathrm{sh}^2(x)$$

$$e^x = \mathrm{sh}(x) + \mathrm{ch}(x)$$

$$x = \ln\left(\mathrm{sh}(x) + \mathrm{ch}(x)\right)$$

$$\int \mathrm{sh}^2(x)dx = \frac{\mathrm{sh}(x)\mathrm{ch}(x)-x}{2} + C$$

$$\int \mathrm{ch}^2(x)dx = \frac{\mathrm{sh}(x)\mathrm{ch}(x)+x}{2} + C$$

Integration of algebraic functions

Type 1. If the function contains $\sqrt{x^2-a^2}$, then:

$$x = a \cdot \mathrm{ch}(t)$$

$$dx = a \cdot \mathrm{sh}(t)dt$$

$$\sqrt{x^2-a^2} = a \cdot \mathrm{sh}(t)$$

Type 2. If the function contains $\sqrt{x^2+a^2}$, then:

$$x = a \cdot \mathrm{sh}(t)$$

$$dx = a \cdot \mathrm{ch}(t)dt$$

$$\sqrt{x^2+a^2} = a \cdot \mathrm{ch}(t)$$

Integration of trigonometric functions

$$\sin\alpha\sin\beta = \frac{1}{2}\Big(\cos(\alpha-\beta) - \cos(\alpha+\beta)\Big)$$
$$\cos\alpha\cos\beta = \frac{1}{2}\Big(\cos(\alpha-\beta) + \cos(\alpha+\beta)\Big)$$
$$\sin\alpha\cos\beta = \frac{1}{2}\Big(\sin(\alpha-\beta) + \sin(\alpha+\beta)\Big)$$

If a function is made up of trigonometric functions but is not similar to above formulas, then we need to use substitution:

$$t = \tan\frac{x}{2}$$
$$x = 2\arctan t$$
$$dx = \frac{2dt}{1+t^2}$$
$$\sin x = \frac{2t}{1+t^2}$$
$$\cos x = \frac{1-t^2}{1+t^2}$$

Definite integrals

- $\int_a^b f(x)dx = F(b) - F(a)$,
- area $\int_a^b f(x)dx$,
- volume $\pi\int_a^b (f(x))^2 dx$,
- side-surface $2\pi\int_a^b |f(x)|\sqrt{1+(f'(x))^2}dx$,
- complete surface $2\pi\int_a^b |f(x)|\sqrt{1+(f'(x))^2}dx + \pi f^2(a) + \pi f^2(b)$,
- length of the curve $\int_a^b \sqrt{1+(f'(x))^2}dx$.