Exercise 1. Calculate double integrals over a rectangle R.

a) $\iint_R xy\sqrt{1+x^2+y^2}\ dxdy$ $R: 0 \le x \le 1,\ 0 \le y \le 1$ *answer:* $\frac{1-8\sqrt{2}+9\sqrt{3}}{15}$

b) $\iint_R \frac{1}{(x+y+1)^3}\ dxdy$ $R: 0 \le x \le 2,\ 0 \le y \le 1$ *answer:* $\frac{5}{24}$

c) $\iint_R x\sin(xy)\ dxdy$ $R: 0 \le x \le 1,\ \pi \le y \le 2\pi$ *answer:* 0

d) $\iint_R (2x-3y^2)\ dxdy$ $R: -1 \le x \le 1,\ 0 \le y \le 2$ *answer:* -16

e) $\iint_R x\cos(x^2+y)\ dxdy$ $R: -\sqrt{\pi} \le x \le 0,\ 0 \le y \le \pi$ *answer:* 2

f) $\iint_R x^2ye^{xy}\ dxdy$ $R: 0 \le x \le 1,\ 0 \le y \le 2$ *answer:* $\frac{e^2-1}{4}$

Exercise 2. Express the following double integrals over a rectangle R as products of single integrals and evaluate them.

a) $\iint_R \frac{x^2}{1+y^2}\ dxdy$ $R: 0 \le x \le 1,\ 0 \le y \le 1$ *answer:* $\frac{\pi}{12}$

b) $\iint_R \frac{x}{y}\ dxdy$ $R: 0 \le x \le 2,\ 1 \le y \le e$ *answer:* 2

c) $\iint_R e^{x-y}\ dxdy$ $R: -1 \le x \le 1,\ -1 \le y \le 1$ *answer:* $e^2+\frac{1}{e^2}-2$

d) $\iint_R xy(x^2+y^2)\ dxdy$ $R: 0 \le x \le 1,\ 0 \le y \le 1$ *answer:* $\frac{1}{4}$

e) $\iint_R \cos(x+y)\ dxdy$ $R: -\frac{\pi}{4} \le x \le \frac{\pi}{4},\ 0 \le y \le \frac{\pi}{4}$ *answer:* 1

f) $\iint_R xy\ln\frac{x}{y}\ dxdy$ $R: 1 \le x \le e,\ 1 \le y \le 2$ *answer:* $e^2(\frac{3}{4}-\ln 2)+\ln 2$

Exercise 3. Write down the formula for $\iint_D f(x,y)\ dxdy$ as an iterated double integral if D is a region bounded by the following curves. Sketch the region in each case.

a) $x=0,\ y=0,\ x^2+y^2=25,\ (x,\ y \ge 0)$ *answer:* $\int_0^5 \left(\int_0^{\sqrt{25-x^2}} f(x,y)\ dy \right) dx$

b) $y=\frac{1}{x},\ y=\sqrt{x},\ x=2$ *answer:* $\int_1^2 \left(\int_{\frac{1}{x}}^{\sqrt{x}} f(x,y)\ dy \right) dx$

c) $x^2+y=2,\ y^3=x^2,\ x=0$ *answer:* $\int_0^1 \left(\int_{x^{\frac{2}{3}}}^{2-x^2} f(x,y)\ dy \right) dx$

Exercise 4. Reverse the order of integration in the following integrals. Evaluate both integrals. What is the geometric representation of the integrals? Sketch the region in each case.

a) $\int_0^1 dx \int_0^{\ln x} 1\ dy$ *answer:* $\int_0^1 dy \int_{e^y}^{e} 1\ dx = 1$

b) $\int_0^1 dy \int_{y^2}^{\sqrt{y}} 1\ dx$ *answer:* $\int_0^1 dx \int_{x^2}^{\sqrt{x}} 1\ dy = \frac{1}{3}$

c) $\int_0^{\frac{\pi}{2}} dx \int_{\sin x}^{2} 1\ dy$ *answer:* $\int_1^2 dy \int_0^{\frac{\pi}{2}} 1\ dx + \int_0^1 dy \int_0^{\arcsin y} 1\ dx = \pi - 1$

Remark: Divide region in example (c) into two parts.

Exercise 5. Calculate double integrals over regions bounded by given curves. Sketch the region in each case.

a) $\iint\limits_D x^2y\; dxdy$ $\quad D: y=-\sqrt{x},\ y=\frac{1}{x},\ x=1,\ x=2$ $\quad$ *answer:* $-\frac{11}{8}$

b) $\iint\limits_D xy\; dxdy$ $\quad D: y=-x^2+4,\ y=3\sqrt{x},\ y=0$ $\quad$ *answer:* $\frac{19}{12}$

c) $\iint\limits_D (x^2+y)\; dxdy$ $\quad D: y=x^2,\ y^2=x$ $\quad$ *answer:* $\frac{33}{140}$

d) $\iint\limits_D \frac{x^2}{y^2}\; dxdy$ $\quad D: y=\frac{1}{x},\ y=x,\ x=2$ $\quad$ *answer:* $\frac{9}{4}$

e) $\iint\limits_D 1\; dxdy$ $\quad D: y=x^2,\ y=4-x^2$ $\quad$ *answer:* $\frac{16\sqrt{2}}{3}$

f) $\iint\limits_D 2x\; dxdy$ $\quad D: y^2=x+2,\ y=-x,\ x=2$ $\quad$ *answer:* $\frac{182}{15}$

g) $\iint\limits_D 2xy\; dxdy$ $\quad D: y=x^2,\ y=2+|x|$ $\quad$ *answer:* 0

h) $\iint\limits_D (x+2y)\; dxdy$ $\quad D: y=-\sqrt{x},\ y=-2\sqrt{x},\ 1\le x\le 4$ $\quad$ *answer:* $-\frac{101}{10}$

Exercise 6. Calculate double integrals over regions bounded by given curves using polar coordinates. Sketch the region in each case.

a) $\iint\limits_D (x^2+y^2)\; dxdy$ $\quad D: x^2+y^2\le 4$ $\quad$ *answer:* 8π

b) $\iint\limits_D \sqrt{x^2+y^2-9}\; dxdy$ $\quad D: x^2+y^2\le 25,\ x^2+y^2\ge 9$ $\quad$ *answer:* $\frac{128\pi}{3}$

c) $\iint\limits_D (x+y)\; dxdy$ $\quad D: x^2+y^2\le 2x$ $\quad$ *answer:* $-\pi$

d) $\iint\limits_D (x+y)\; dxdy$ $\quad D: x^2+y^2\le x+y$ $\quad$ *answer:* $-\frac{\pi}{2}$

e) $\iint\limits_D x^2\sqrt{x^2+y^2}\; dxdy$ $\quad D: x^2+y^2\le 1,\ y\ge x$ $\quad$ *answer:* $\frac{\pi}{10}$

f) $\iint\limits_D \frac{1}{\sqrt{x^2+y^2}}\; dxdy$ $\quad D: x^2+y^2\ge 1,\ x^2+y^2\le 4,\ y\ge x$ $\quad$ *answer:* π

Exercise 7. Using a double integral, calculate the area of regions given in **Exercise 3**, **Exercise 5** and **Exercise 6**.

Exercise 8. Calculate the volume of a solid bounded by the following curves. Sketch the solid in each case.

a) $y=x^2,\ y=1,\ z=0,\ z=2y$ $\quad$ *answer:* $\frac{8}{5}$

b) $x=0,\ y=0,\ x+y=1,\ z=0,\ z=2xy$ $\quad$ *answer:* $\frac{1}{12}$

c) $x=0,\ y=1,\ 2x+y=5,\ z=0,\ z=xy$ $\quad$ *answer:* $\frac{113}{96}$

Exercise 9. Calculate the volume of a solid with base bounded by curves $y^2=x+2$, $y=-x$, $x=2$ having height equal 9. Sketch the solid. *answer:* $\frac{111}{2}$.

Exercise 10. Calculate the volume of a solid with base bounded by a curve $x^2+y^2=5$. The base of the solid lies on the OXY plane. The solid is bounded from above by plane x^2+y^2. Sketch the solid. *answer:* $\frac{625\pi}{2}$.

Exercise 11. Calculate the surface of a plane $2x+2y+z=8$ bounded by the coordinate system axes. Sketch a diagram. *answer:* 24.