Exercise 1. Find minima, maxima and saddle points of the following functions. Plot each function in the surrounding of the critical points using the 3D function plotter.

$$a(x,y) = 3x^3 + 3x^2y - y^3 - 15x, \qquad b(x,y) = e^{x-y}(x^2 - 2y^2),$$
$$c(x,y) = e^{2x}(x + y^2 + 2y), \qquad d(x,y) = x^3 - 4xy + 2y^2,$$
$$e(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2, \qquad f(x,y) = x^3 + y^3 - 3xy,$$
$$g(x,y) = 2x^3 + xy^2 + 5x^2 + y^2, \qquad h(x,y) = x^3 + 8y^3 - 6xy + 5,$$
$$i(x,y) = xy\ln(x^2 + y^2), \qquad j(x,y) = x^3 + 3xy^2 - 15x - 12y.$$

Remark: Find minima, maxima and saddle points of functions i and j only for $(x,y) \in [0,\infty) \times [0,\infty)$. For function i, notice that $\ln(x^2 + y^2) = 0$ for $(x,y) = (\frac{1}{\sqrt{2e}}, \frac{1}{\sqrt{2e}})$.

Exercise 2. Find the largest values, the smallest values and the saddle points of the following functions in a given region. Plot each function in the surrounding of the given region using the 3D function plotter.

$a(x,y) = x^2y(4 - x - y)$	in a triangle with sides along lines: $x = 0,\ y = 0,\ x + y + 6 = 0$,
$b(x,y) = x^2 + y^2 - xy + x + y$	in a triangle with sides along lines: $x = 0,\ y = 0,\ x + y + 3 = 0$,
$c(x,y) = x^2 + 2xy - 4x + 8y$	in a rectangle with sides along lines: $x = 0,\ x = 1,\ y = 0,\ y = 2$,
$d(x,y) = x^3 + y^3 - 9xy + 27$	in a square with sides along lines: $x = 0,\ x = 1,\ y = 0,\ y = 1$,
$e(x,y) = xy$	in a square given by equation: $\lvert x\rvert + \lvert y\rvert = 1$,
$f(x,y) = xy$	in a circle given by equation: $x^2 + y^2 \leq 1$,
$g(x,y) = 2x^2 - 2y^2$	in a circle given by equation: $x^2 + y^2 \leq 4$,
$h(x,y) = x + y$	in a circle given by equation: $x^2 + y^2 \leq 4$,
$i(x,y) = x^2 - 2xy + 2y^2 - 2y$	in a region bounded by curves: $y = \frac{1}{2}x^2,\ y = 2$.

Exercise 3. Using the 3D plotter and a method of trials and errors (or deep thoughts ;)), find a function that:

a) has infinitely many mimima and maxima,

b) has infinitely many saddle points,

c) has a saddle point and a minimum,

d) has a saddle point and a maximum,

e) has a minimum and a maximum.

Verify your finding by applying the minimum-maximum test.

Remark: When looking for a function that has infinitely many minima, maxima or saddle points, it is worthwhile to try out combinations of polynomials and trigonometric functions.