Exercise 1. Calculate triple integrals over a region R. Sketch each region.

a) $\iiint_R (x^2+y^2+z^2)\ dxdydz$ $R=[0,1]\times[0,2]\times[0,3]$ *answer:* 28

b) $\iiint_R xy\sin z\ dxdydz$ $R=[\frac{1}{6},\frac{1}{2}]\times[0,1]\times[0,\pi]$ *answer:* $\frac{1}{9}$

c) $\iiint_R z2^{x-y}\ dxdydz$ $R=[0,1]\times[0,1]\times[-2,1]$ *answer:* $\frac{-3}{2\ln 2\ln 4}$

d) $\iiint_R \cos(x+y+z)\ dxdydz$ $R=[0,\pi]\times[0,\pi]\times[0,\pi]$ *answer:* 0

Exercise 2. Calculate the following integrals over regions bounded by given planes. Sketch each region.

a) $\iiint_R (x+y)\ dxdydz$ $R: y=x^2,\ y=1,\ z=0,\ z=2$ *answer:* $\frac{8}{5}$

b) $\iiint_R (x^2+y^2)\ dxdydz$ $R: x=0,\ y=0,\ z=0,\ x+y+z=3$ *answer:* $\frac{81}{10}$

c) $\iiint_R y\ dxdydz$ $R: x^2+y^2=3,\ z=-1,\ z=2$ *answer:* $6\sqrt{3}$

d) $\iiint_R (x+y+2z)\ dxdydz$ $R: x^2+z^2=4,\ y=2,\ y=3$ *answer:* 10π

Exercise 3. Calculate the volume of regions given in **Exercise 2**.

Answers: (a) $\frac{9}{2}$*, (b)* $\frac{8}{3}$*, (c)* 9π*, (d)* 4π*.*

Exercise 4. Calculate the mass of solid V bounded by given planes and having density ρ. Sketch each solid.

a) $x=0,\ x=1,\ y=0,\ y=3,\ z=0,\ z=2$ $\rho(x,y,z)=x+y+z$ *answer:* 18

b*) $x^2+y^2=4,\ z=0,\ z=2\sqrt{x^2+y^2}$ $\rho(x,y,z)=z$ *answer:* 16π

c) $x=0,\ y=0,\ z=0,\ x+y=1,\ z=x+2y$ $\rho(x,y,z)=3+2x+2y-2z$ *answer:* $\frac{1}{6}$

Exercise 5. Calculate the following integrals using cylindrical coordinates.

a) $\iiint_R (x^2+y^2)\ dxdydx$ $R: \sqrt{x^2+y^2}\le z\le 1$ *answer:* $\int_0^{2\pi}\left(\int_0^1\left(\int_r^1 r^2\cdot r\ dh\right)\ dr\right)\ d\theta=\frac{\pi}{10}$

b) $\iiint_R x^2\ dxdydx$ $R: 0\le z\le 9-x^2-y^2$ *answer:* $\int_0^{2\pi}\left(\int_0^3\left(\int_0^{9-r^2} r^2\cdot\cos^2\theta\cdot r\ dh\right)\ dr\right)\ d\theta=\frac{243\pi}{4}$

Exercise 6. Calculate the following integrals using spherical coordinates.

a) $\iiint_R (x^2+y^2+z^2)\ dxdydx$ $R: -\sqrt{4-x^2-y^2}\le z\le 0$ *answer:* $\frac{64\pi}{5}$

b) $\iiint_R z^2\sqrt{x^2+y^2+z^2}\ dxdydx$ $R: 0\le z\le\sqrt{4-x^2-y^2},\ x\ge 0,\ y\ge x$ *answer:* $\frac{8\pi}{9}$