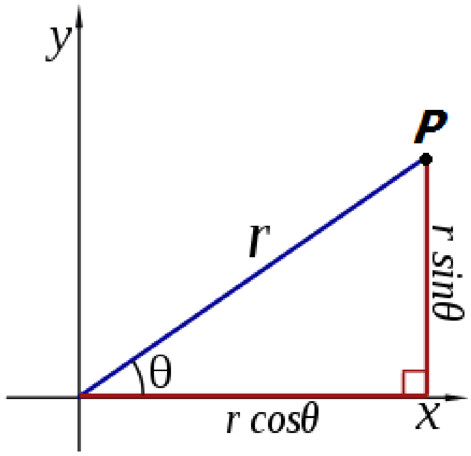
POLAR COORDINATES

The polar coordinate system is a two-dimensional coordinate system in which each point on a plane is determined by a distance from a fixed point and an angle from a fixed direction.



The polar coordinates of point P are

$$P = (r\cos\theta, r\sin\theta)$$

APPLICATIONS

Polar coordinates may be used to show, that a limit (with $(x, y) \to (0, 0)$) exists or does not exist. The basic way to do that is:

1. Substitute polar coordinates for x and y.
2. Simplify the resulting expression – remember that if $(x, y) \to (0, 0)$, then $r \to 0$ and θ is arbitrary.
3. Remember, that the following limits do not exist: $\lim\limits_{r\to\ldots} sin\theta$, $\lim\limits_{r\to\ldots} cos\theta$, $\lim\limits_{r\to\ldots} tan\theta$, $\lim\limits_{r\to\ldots} cot\theta$ – because they depend only on θ!

Example 1. Show that $\lim\limits_{(x,y)\to(0,0)} \frac{x}{y}$ does not exist.
Solution: $\lim\limits_{(x,y)\to(0,0)} = \lim\limits_{r\to 0} \frac{r\cos\theta}{r\sin\theta} = \lim\limits_{r\to 0} \frac{\cos\theta}{\sin\theta} = \lim\limits_{r\to 0} \cot\theta$ – this limit does not exist, because $\cot\theta$ may take on different values.

Example 2. Show that $\lim\limits_{(x,y)\to(0,0)} \frac{3yx}{x^2+y^2}$ does not exist.
Solution: $\lim\limits_{(x,y)\to(0,0)} \frac{3yx}{x^2+y^2} = \lim\limits_{r\to 0} \frac{3r^2\sin\theta\cos\theta}{r^2(\cos^2\theta+\sin^2\theta)} = \lim\limits_{r\to 0} \frac{3\sin\theta\cos\theta}{1} = \lim\limits_{r\to 0} \frac{3}{2}\sin 2\theta$ – again, this limit depends only on θ, so it does not exist.

Example 3. Calculate $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2}$.
Solution: $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2} = \lim\limits_{r\to 0} \frac{r^4\cos^2\theta\sin^2\theta}{r^2(\cos^2\theta+\sin^2\theta)} = \lim\limits_{r\to 0} r^2\cos^2\theta\sin^2\theta$.
We know that $\cos^2\theta\sin^2\theta \in [0, 1]$, so finally $\lim\limits_{r\to 0} r^2\cos^2\theta\sin^2\theta = 0$.