

## EXERCISES

1. Put the following numbers in a nondecreasing order without using a calculator:

$$2^2, 2^{(2^2)}, 2^{\sqrt{2}}, 2^{\frac{1}{2}}, 2^{(\frac{1}{2})^2}, 2^{(2^{0.5})}, (2^2)^2.$$

2. Solve the following equations:

(a)  $5^x = 0.008$

(b)  $3^{\frac{5}{4}-4x} = \left(\frac{1}{3}\right)^{3x^2}$

(c)  $2^x - 2^{x-4} = 15$

(d)  $6^{x-5} \cdot 36^{3-2x} = 4^{3x-3}$

(e)  $\sqrt[3]{3} \cdot \left(\frac{1}{27}\right)^{\frac{1-x}{3}} + 9^{\frac{x}{2}} = 3 + \sqrt[3]{3}$

(f)  $2^{2x} + 2^x = 20$

(g)  $\sqrt{(0.25)^{5-\frac{x}{4}}} = 2^{\sqrt{x+1}-4}$

(h)  $3^{x+2} + 9^{x+1} = 810$

(i)  $\left(\frac{5}{2}\right)^{\sqrt{9-x}-1} = 0.4^{\frac{4+\sqrt{9-x}}{\sqrt{9-x}}-5}$

(j)  $2^{2x} \cdot 9^x - 2 \cdot 6^{3x-1} + 4^{2x-1} \cdot 3^{4x-2} = 0$

(k)  $2 \cdot 3^{x+1} - 4 \cdot 3^{x-2} = 450$

(l)  $3^{|x+1|+1} - 5 \cdot 3^{|x+1|-1} = 12$

(m)  $2^x + 2^{x+1} + 2^{x+2} = 6^x + 6^{x+1}$

(n)  $6 \cdot 9^x + 5 \cdot 6^x - 6 \cdot 4^x = 0$

(o)  $\sqrt{2^x} \cdot \sqrt{3^x} = 6^x - 30$

(p)  $4^{\sqrt{x-1}} + 1 - 4^{\frac{2\sqrt{x-1}-1}{2}} = 7$

(q)  $\frac{10^x + 10^{-x}}{10^x - 10^{-x}} = 5$

(r)  $8(4^x + 4^{-x}) - 54(2^x + 2^{-x}) + 101 = 0$

(s)  $-6 + 3^{5x} + 3^{5x-1} + 3^{5x-2} + \dots = \sqrt{7 \cdot 3^{5x} + 11}$

(t)  $(3 - 2\sqrt{2})^x + (3 + 2\sqrt{2})^x = 6$

(u)  $(2 + \sqrt{3})^x + (2 - \sqrt{3})^x = 4$

3. Solve the following inequalities:

(a)  $\left(\frac{7}{11}\right)^{7x-11} \geq \left(\frac{11}{7}\right)^{11x-7}$

(b)  $\frac{3}{10} \cdot \left(\frac{3}{2}\right)^{x-2} < \frac{6}{5} \cdot \left(\frac{3}{2}\right)^{x-3} - \frac{1}{2}$

(c)  $\left(\frac{1}{2}\right)^{2x^2+x-1} > \left(\frac{1}{4}\right)^{0.5x^2+x-0.125}$

(d)  $2^{2x+4} - 4^x > 15$

(e)  $4^{x+4} < 4^{1-x}$

(f)  $\left(\frac{1}{3}\right)^{2x} - 12 \cdot \left(\frac{1}{3}\right)^x + 27 > 0$

(g)  $7^{-x} - 3 \cdot 7^{x+1} > 4$

(h)  $2 \cdot 3^x \leq 3 + \sqrt{5 - 2 \cdot 3^x}$

(i)  $3^{3x+1} - 4 \cdot 27^{x-1} + 9^{1.5x-1} < 80$

(j)  $5 \cdot 4^x + 2 \cdot 25^x \leq 7 \cdot 10^x$

(k)  $-2 + 2^{3x} + 2^{3x-1} + 2^{3x-2} + \dots \geq \sqrt{2^{3x} + 2}$

(l)  $5^x - 20 > 5^{3-x}$

(m)  $\left(\frac{1}{3}\right)^{|x-2|} \leq \frac{1}{9}$

(n)  $2^{2x} \leq 3 \cdot 2^{x+\sqrt{x}} + 4 \cdot 2^{2\sqrt{x}}$

(o)  $\frac{25}{5^x} - (5^{\sqrt{x}})^{\sqrt{8x-1}} < 0$

## References

[1] *Matematyka – podstawy z elementami matematyki wyższej*, edited by B. Wikieł, PG publishing house, 2009.