EXERCISES

1. If $f(x) = 4x^2 - x + 1$ find $f(3), f(-3), f(v), f(-v), f(b+1), 2f(b), f(2a), f(a^2), [f(a)]^2$

2. Find the domain and the zeros of the function

(a) $f(x) = \dfrac{x}{x^2 - 16}$

(b) $f(x) = \dfrac{x+4}{x^2 - 16}$

(c) $f(x) = \sqrt{3-x} + \sqrt{x+5}$

(d) $f(x) = \dfrac{5x^3 + 2}{\sqrt{x^2 - x - 6}}$

3. Find the composite functions $f(g(x))$, $g(f(x))$ and their domains.

(a) $f(x) = 2x + 1$, $g(x) = 3x - 2$

(b) $f(x) = \dfrac{4}{x+1}$, $g(x) = 2x + 4$

(c) $f(x) = \sqrt{x}$, $g(x) = x + 2$

(d) $f(x) = \sqrt{x}$, $g(x) = x^2 + 2$

(e) $f(x) = \dfrac{2}{x}$, $g(x) = \dfrac{3}{x-2}$

4. Suppose the graph of f is given. Write equations for graphs that are obtained from the graphs of f as follows

(a) Shift 2 units to the right

(b) Reflect bout the $x-$ axis

(c) Shift 2 units upward

(d) Stretch vertically by a factor of 2

(e) Shift 2 units downward

(f) Stretch horizontally by a factor of 2

(g) Reflect about the $y-$axis

(h) Shift 2 units to the left

5. The graph of $y = f(x)$ is given. Match each equation with its graph

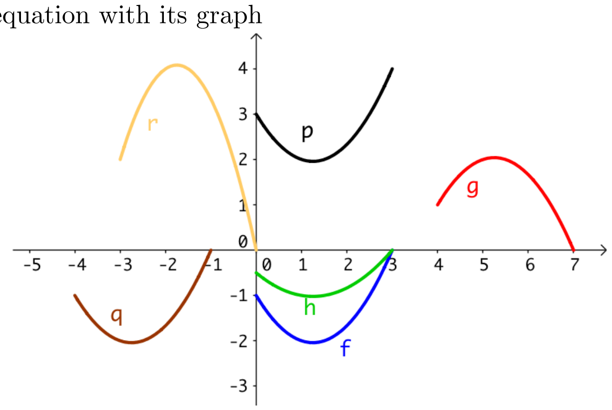
(a) $y = f(x + 4)$

(b) $y = \frac{1}{2}f(x)$

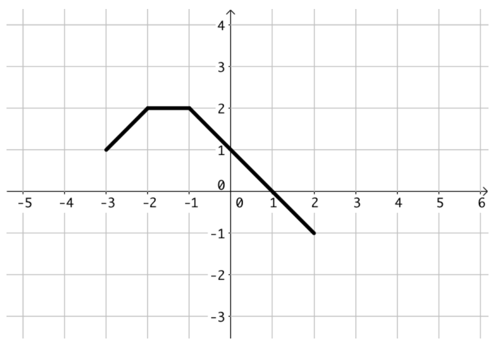
(c) $y = f(x) + 4$

(d) $y = -f(x - 4)$

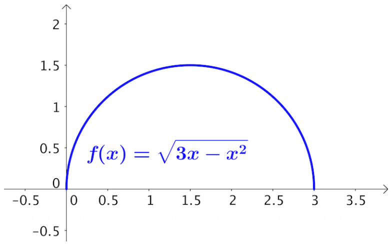
(e) $y = -2f(x + 3)$

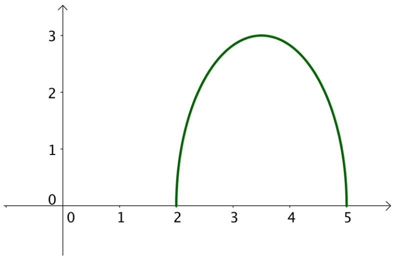
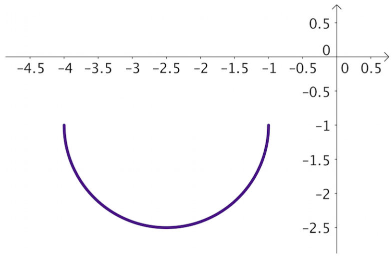


6. The graph of f is given. Draw the graphs of $y = f(x-2) + 2$, $y = f(|x|) - 1$, $y = |f(x-1)|$



7. The graph of $f(x)=\sqrt{3x-x^2}$ is given. Use transformations to create functions whose graphs are as shown.



8. (a) Use a graphing software (e.g. Geogebra) to sketch the graph of $y=\sqrt{x}$. Then use transformations to graph $y=\sqrt{x}-3,\ y=\sqrt{x-3},\ y=-\sqrt{x},\ y=\sqrt{-x}, y=\sqrt{|x|}$

(b) Start with the graph of $y=\dfrac{1}{x^2+1}$, and then use transformations to graph $y=\dfrac{1}{(x+1)^2+1}-2$

9. Determine whether f is odd, even, or neither.

(a) $f(x)=\dfrac{x^2}{x^4+x}$

(b) $f(x)=x^6+x^2+\dfrac{x}{x^7+x}$

(c) $f(x)=x^3\sqrt{x^4+1}$

(d) $f(x)=|5-4x|-|4x+5|$

(e) $f(x)=(x+x^3)\cdot\operatorname{sgn}x$

10. Can you think of a function $f\ :\ I\!R\to I\!R$ that is both even and odd ? If yes, what is the function ? If no, show that such function does not exist.

11. Show that

(a) the sum and difference of two even (odd) functions with the same domains is an even (odd) function.

(b) the product of two even (odd) functions with the same domain is an even function.

(c) a composition of two one-to-one functions is also a one-to-one function.

(d) a strictly monotonic function is a one-to-one function.

12. Which of these functions are 1-1 and which ones are "onto"?

- $f(x)=4x-2,\ f:I\!R\to I\!R,$
- $f(x)=5,\ f:I\!R\to\{5\},$
- $f(x)=\frac{1}{x^2},\ f:I\!R_+\to I\!R_+,$
- $f(x)=2\operatorname{sgn}x,\ f:I\!R\to\{-2,0,2\}$
- $f(x)=\lfloor x\rfloor,\ f:I\!R\to\mathbb{Z}$
- $f(x)=x^2-3x,\ f:\mathbb{Z}\to\mathbb{Z},$
- $f(x)=|x-1|+2,\ f:\mathbb{N}\to\mathbb{N}$

13. Give your own example of a function (both the graph and the formula), which:

- is 1-1, but not "onto",
- is "onto", but not 1-1,
- is 1-1 and "onto",
- is neither 1-1 nor "onto".

14. Sketch a graph of a function that has the following properties

(a) $D_f=I\!R\setminus\{0\},\ R_f=(-2,1)$, is increasing

(b) $f:I\!R\to I\!R$, is not onto, odd, and not monotonic

(c) $f:I\!R\to(1,\infty)$, is a bijection

(d) $f : \mathbb{R} \to \langle 1, \infty)$, is a bijection

15. Find inverse functions. Plot f and f^{-1}:

(a) $f(x) = 2x + 3$

(b) $f(x) = 4 - x^2$, $D_f = \langle 0, \infty)$

References

[1] *Matematyka – podstawy z elementami matematyki wyszej*, edited by B. Wikieł , PG publishing house, 2009.