Exercises

1. Fill in the missing numbers:

(a) $\log_4 1 = \square$

(b) $\log 1000 = \square$

(c) $\log \square = -0.5$

(d) $\log_2 \frac{1}{8} = \square$

(e) $\ln \sqrt{e^3} = \square$

(f) $\log_{0.5} 32 = \square$

(g) $\log_4 4 = \square$

(h) $\log 0.001 = \square$

(i) $\log_5 0.04 = \square$

(j) $\log_{\square} 10 = 0.5$

(k) $\log_{\square} 9 = 2$

(l) $\log_2 \square = -2$

(m) $\log_2 8 = \square$

(n) $\ln e^2 = \square$

(o) $\log_3 \square = 3$

2. Determine the signs of the following expressions without using a calculator

(a) $\ln(e-2)$, (b) $\log_{\frac{1}{e}}(e-1)$, (c) $\log 5$, (d) $-1+\log 5$, (e) $\log 0.2$, (f) $\log_3(\sqrt{2}-1)$.

3. Evaluate without using a calculator:

(a) $\log_{\sqrt{2}} 16$
(b) $\log_4 0.5$
(c) $\log_{\frac{2}{3}} 2,25$
(d) $\log_{\frac{1}{9}} 3\sqrt[3]{3}$
(e) $\log_{\frac{1}{3}} \frac{3}{\sqrt[3]{9}}$
(f) $16^{\log_2 3}$
(g) $10^{2+2\log 7}$
(h) $(\sqrt[3]{4})^{\frac{3}{2\log_3 2}}$
(i) $1000^{\frac{1}{3}\cdot\log \sqrt[3]{3}}$
(j) $\log_9 5 \cdot \log_{25} 27$
(k) $3^{\log_6 4+2\log_6 3}$
(l) $\left(\frac{1}{x}\right)^{\log_x 8}$
(m) $10 \cdot 10^{0.5\log 9-\log 2}$

4. Determine the domains and plot the graphs of the following functions

a) $f(x) = |-\log_3(x+1)+1|-1$, b) $f(x) = -|\log_{\frac{1}{2}}(x-2)-2|$, c) $f(x) = |\ln|x||$.

5. Find the domains of the following functions

(a) $y = \ln(6-x-x^2)$
(b) $y = \log_x(6-x-x^2)$
(c) $y = \log_{x^2}(4-x^2)$
(d) $f(x) = \sqrt{\log_{x-2}(x^2-8)-1}$
(e) $f(x) = \sqrt{2-\log_{(x^3+1)}\cdot\log_{(x+1)} x}$

6. Determine the domain, range, and find the inverse function

(a) $f(x) = 2+3^{2x+1}$
(b) $f(x) = 2\cdot\log_4(5x-1)$
(c) $f(x) = 2^{2x+2}$
(d) $f(x) = \log_5(2x+1)-3$
(e) $f(x) = \frac{1}{3}\cdot 3^{x+1}-3$
(f) $f(x) = \log_{0.5}(2+x)-0.5$

7. Solve the equations

(a) $e^{2x} = 5$

(b) $\ln(5x - e) = 1$

(c) $\log_3(4 \cdot 3^{x-1} - 1) = 2x - 1$

(d) $\log(5x^2 + 2x - 1) - \log(x + 2) = 1$

(e) $\log_{2x+1} \frac{2}{3} = -1$

(f) $\frac{\log 2x}{\log(4x - 15)} = 2$

(g) $\log_4 x + \log_4(12 - 2x) = 2$

(h) $\sqrt{2 \log x - 7} = 5 - \log x$

(i) $\frac{1}{2} \log(2x + 7) + \log \sqrt{7x + 5} = 1 + \log \frac{9}{2}$

(j) $\log_3 x + \log_5 x = \frac{\log 15}{\log 3}$

(k) $\log_x 2 + \log_2 x = 2.5$

(l) $x^{\log x} = 10$

(m) $10^{\log^2 x} + x^{\log x} = 20$

(n) $\log^2 x + 2 \log(10x) = 17$

(o) $\log_2(\log_{0.5}(\log_2 x)) = 0$

(p) $\log_x(5x^2) \cdot (\log_5 x)^2 = 1$

(q) $-2 + \log_4(3x) + \log_4 \sqrt[4]{3x} + \log_4 \sqrt[8]{3x} + \cdots = \sqrt{\log_4(3x)^2}$

(r) $\log(2 - x) - \log^2(2 - x) + \log^3(2 - x) - \log^4(2 - x) + \cdots = -1$

8. Solve the inequalities

(a) $\log_{\frac{1}{3}} x > -1$

(b) $\log_2(x + 1) > 3$

(c) $\log_x 4 < 2$

(d) $\log_{0.5} \frac{2x + 1}{3x + 2} > 3$

(e) $\log(x - 3) - \log(27 - x) \leq -\log 5 - 1$

(f) $\log_{\frac{1}{3}}(|x| - 1) > -2$

(g) $8^{\log_2 x} - 2x^2 \geq x - 2$

(h) $x^{2 \log_5 6} - 13 \cdot 6^{\log_5 x} + 42 \leq 0$

(i) $\log_3 \left[\log_4(x^2 - 5)\right] > 0$

(j) $3^{\log_{\frac{1}{5}}(x^2 - 4x - 4)} < 1$

(k) $\sqrt{7 - \log_2 x} < \log_2 x - 5$

(l) $\log_{\frac{1}{\sqrt{5}}}(6^{x+1} - 36^x) \geq -2$

(m) $\log_{x^2}(x + 6) \geq 1$

(n) $\sqrt{8 - \log_2 x} \geq \log_2 x - 6$

(o) $\log_{16} x + \log_4 x + \log_2 x < 7$

(p) $|\log|x - 1| + 1| \geq 2$

(q) $\frac{1}{\log x} + \frac{1}{1 - \log x} > 1$

(r) $3 \geq \log_{(x+1)}(x - 1) \cdot \log_{(x-1)}(2x^2 + 2x + 1)$

References

[1] *Matematyka – podstawy z elementami matematyki wyszej*, edited by B. Wikieł , PG publishing house, 2009.