Exercises

1. Find the domains of the following functions:

(a) $f(x) = x\sqrt[3]{(x-2)^5} + x^3(3-x)^{\frac{7}{4}}$

(b) $y = (x-2)^{\frac{3}{2}}(x-3)^{\frac{2}{3}} - (4-x)^{\frac{5}{4}}(5-x)^{\frac{4}{5}}$

(c) $y = x^2(x^2-5x)^{-\frac{5}{3}} + \sqrt{1-\dfrac{1}{x-1}}$

(d) $y = x^{15}(5-x)^{\sqrt{2}}$.

2. Sketch the graphs of the following functions

(a) $f(x) = x^{3/2}$
(b) $f(x) = x^{2/3}$
(c) $f(x) = x^{8/6}$
(d) $f(x) = x^{6/8}$
(e) $f(x) = x^{-5/4}$
(f) $f(x) = x^{-4/12}$
(g) $f(x) = x^{\pi/3}$
(h) $f(x) = x^{\pi/5}$
(i) $f(x) = x^{-e}$

3. Solve the following equations

(a) $\sqrt{x+3} = 3$
(b) $(x^2-9)\sqrt{2-x} = 0$
(c) $\sqrt{5x+10} = 8-x$
(d) $x - \sqrt{x+1} = 5$
(e) $\sqrt{4+2x-x^2} = x-2$
(f) $x + \sqrt{10x+6} = 9$
(g) $\sqrt{x} + 9 = 4\sqrt{x}$
(h) $\sqrt{2x-3} + \sqrt{4x+1} = 4$
(i) $3 - \sqrt{x-1} = \sqrt{3x-2}$
(j) $\sqrt{x-3} = 3 - \sqrt{x}$
(k) $\sqrt{9x-17} = 3\sqrt{x-4} + 1$
(l) $\sqrt{x-2} - \sqrt{x-9} = 1$
(m) $\sqrt{x+13} - \sqrt{x-3} = 2$
(n) $\sqrt{x-9} = \dfrac{36}{\sqrt{x-9}} - \sqrt{x}$

4. Solve the following inequalities:

(a) $\sqrt{11-x} > x-9$
(b) $\sqrt[3]{x-2} < -3$
(c) $\sqrt{8-x} < \dfrac{10-x}{7}$
(d) $\sqrt{x-2} + x \geq 4$
(e) $\sqrt{x^2-4x} > x-3$
(f) $4\sqrt{10+x} > 13+x$
(g) $\sqrt{x+3} + \sqrt{3x-2} \leq 7$
(h) $\dfrac{\sqrt{x+20}}{x} < 1$
(i) $\sqrt{(x-6)(1-x)} \leq 2x+3$
(j) $x - 1 < \sqrt{7-x}$
(k) $\sqrt{4x-x^2} > x-2$.
(l) $\sqrt{10+x} + \sqrt{10-x} < 6$
(m) $\sqrt{5x^2+10x+1} \geq 7-2x-x^2$

References

[1] *Matematyka – podstawy z elementami matematyki wyszej*, edited by B. Wikieł , PG publishing house, 2009.