How to use Gaussian elimination to solve a system of equations

My example is:

$$\begin{cases} 4x_1 - x_2 + 2x_3 + x_4 = 1 \\ 2x_1 + 3x_2 - x_3 - 2x_4 = 2 \\ \quad + 7x_2 - 4x_3 - 5x_4 = 3 \\ 2x_1 - 11x_2 + 7x_3 + 8x_4 = 2 \end{cases}$$

I can rewrite the above data as three matrices:

$$A = \begin{bmatrix} 4 & -1 & 2 & 1 \\ 2 & 3 & -1 & -2 \\ 0 & 7 & -4 & -5 \\ 2 & -11 & 7 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

I have to transform matrix $[A|B]$ into $[I \vdots \begin{smallmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{smallmatrix}]$ using elementary operations on matrices:

possibly some extra columns.

$$\left[\begin{array}{cccc|c} \textcircled{4} & -1 & 2 & 1 & 1 \\ 2 & 3 & -1 & -2 & 2 \\ 0 & 7 & -4 & -5 & 3 \\ 2 & -11 & 7 & 8 & 2 \end{array}\right] \xrightarrow{r_1 \to r_1 - r_4} \left[\begin{array}{cccc|c} 2 & 10 & -5 & -7 & -1 \\ \textcircled{2} & 3 & -1 & -2 & 2 \\ 0 & 7 & -4 & -5 & 3 \\ 2 & -11 & 7 & 8 & 2 \end{array}\right] \xrightarrow{r_2 \to r_2 - r_4} \left[\begin{array}{cccc|c} 2 & 10 & -5 & -7 & -1 \\ 0 & 14 & -8 & -10 & 0 \\ 0 & 7 & -4 & -5 & 3 \\ 2 & -11 & 7 & 8 & 2 \end{array}\right]$$

It's very comfortable to have „1" here. However, if I want to have 1 here, I have to produce fractions. So, instead of making fractions I will only subtract row *4 from row *1.

Let's move to the second row. I want to have 0 here.

Let's divide row two by two, to do not have such big numbers.

$$\left[\begin{array}{cccc|c} 2 & 10 & -5 & -7 & -1 \\ 0 & 7 & -4 & -5 & 0 \\ 0 & 7 & -4 & -5 & 3 \\ 2 & -11 & 7 & 8 & 2 \end{array}\right]$$

We can see that, we have two identical rows, which give us different values.

It's contradiction ↯

because two identical combinations can not give different values!

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