Piotrowski Paweł

My example is : $\begin{cases} x_1 - x_2 + x_3 + 2x_4 - 2x_5 = 1 \\ 2x_1 - x_2 - x_3 + 3x_4 - x_5 = 3 \\ -x_1 - x_2 + 5x_3 - 4x_5 = -3 \end{cases}$

I can rewrite system of equations as three matrices :

$$A = \begin{bmatrix} 1 & -1 & 1 & 2 & -2 \\ 2 & -1 & -1 & 3 & -1 \\ -1 & -1 & 5 & 0 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 3 \\ -3 \end{bmatrix} \qquad C = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

I have to transform matrix [A I B] into [I : $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$] using elementary operations on matrices.

some extra collumns

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & 2 & -2 & 1 \\ 2 & -1 & -1 & 3 & -1 & 3 \\ -1 & -1 & 5 & 0 & -4 & -3 \end{array}\right] \xrightarrow{r_2 : r_2 - 2r_1} \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 2 & -2 & 1 \\ 0 & 1 & -3 & -1 & 3 & 1 \\ -1 & -1 & 5 & 0 & -4 & -3 \end{array}\right] \xrightarrow{r_3 : r_3 + r_1}$$

I want zero here

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & 2 & -2 & 1 \\ 0 & 1 & -3 & -1 & 3 & 1 \\ 0 & -2 & 6 & 2 & -6 & -2 \end{array}\right] \xrightarrow{r_3 : r_3/(-2)} \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 2 & -2 & 1 \\ 0 & 1 & -3 & -1 & 3 & 1 \\ 0 & 1 & -3 & -1 & 3 & 1 \end{array}\right] \xrightarrow{r_1 : r_1 + r_2} \left[\begin{array}{ccccc|c} 1 & 0 & -2 & 1 & 1 & 2 \\ 0 & 1 & -3 & -1 & 3 & 1 \end{array}\right]$$

We can cross-out one row because its the same as the first one

$$\begin{cases} x - 2z + w + v = 2 \\ y - 3z - w + 3v = 1 \end{cases}$$

$$\begin{cases} x = 2 + 2z - w - v \\ y = 1 + 3z + w - 3v \end{cases}$$