How to invert matrices using the Gaussian elimination algorithm.

Example:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 1 & 3 & 2 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix}$$

First of all, I need to rewrite my matrix then follow it by a vertical line and a unit matrix of appropriate dimensions.

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

My goal is to perform a certain number of operations that will produce a matrix $[I_4 \mid A^{-1}]$

$r_3 \to r_3 - r_1$, $r_1 \to r_1 - r_2$; $r_3 \to r_3 - 2r_2$, $r_4 \to r_4 - 3r_2$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -2 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -4 & -1 & -2 & 1 & 0 \\ 0 & 0 & 2 & -5 & 0 & -3 & 0 & 1 \end{array}\right]$$

In the first step I get rid of the three numbers „3" from last row, „1" and „3" from third row.

$r_3 \to r_3 : 2$, $r_4 \to r_4 - 2r_3$; $r_4 \to r_4 \cdot (-1)$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -2 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & -\tfrac{1}{2} & -1 & \tfrac{1}{2} & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & 1 & -1 \end{array}\right]$$

Next step, which I done was get rid of the numbers diffrent to „1" and „2".

$r_3 \to r_3 + 2r_4$, $r_2 \to r_2 - 2r_4$; $r_1 \to r_1 + 2r_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & -1 & 2 & -2 \\ 0 & 1 & 0 & 0 & 2 & -1 & -2 & 2 \\ 0 & 0 & 1 & 0 & -\tfrac{5}{2} & 1 & \tfrac{5}{2} & -2 \\ 0 & 0 & 0 & 1 & -1 & -1 & 1 & -1 \end{array}\right]$$

In the last step I get rid of the every number „2" from my matrices and I get final result.

I_4 $\qquad$ A^{-1}

$$A^{-1} = \begin{bmatrix} -1 & 1 & 2 & -2 \\ 2 & -1 & -2 & 2 \\ -\tfrac{5}{2} & 1 & \tfrac{5}{2} & -2 \\ -1 & 1 & 1 & -1 \end{bmatrix}$$

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