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How to invert matrices using the gaussian elimination algorithm.

Example:

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 2 & 1 & 3 & 3 \\ 1 & 4 & 2 & 2 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

First of all, I need to rewrite my matrix, then follow it by a vertical line and a unit matrix of appropriate dimensions.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 1 & 3 & 3 & 0 & 1 & 0 & 0 \\ 1 & 4 & 2 & 2 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

My goal is to perform a certain number of operations that will produce a matrix:

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & \\ 0 & 0 & 1 & 0 & A^{-1} \\ 0 & 0 & 0 & 1 & \end{array}\right]$$

I_4

I would like to have two zeros insted 2 and 1, because then my first column will be look exactly like in I_4.

$r_2 \to r_2 - 2r_1$

$r_3 \to r_3 - r_1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 4 & 1 & 2 & -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_3 \to r_3 - 2r_4} \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & -2 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_1 \to r_1 - r_3} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & 0 & -1 & 2 \\ 0 & 1 & 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & -2 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

I would like to have zero here.

I would like to have zero here to.

I would like to have also zero here, but than my last column will be look like in I_4.

$r_2 \to r_2 - 3 \cdot r_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & 0 & -1 & 2 \\ 0 & -5 & 1 & 0 & -2 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & -2 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

If I get rid of this number, I will three columns which look like exactly in I_4.

$r_2 \to r_2 - r_3$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & 0 & -1 & 2 \\ 0 & -5 & 0 & 0 & -1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & -2 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

I need here number 1, so I must divided second row by ~~5~~.

$r_2 \to r_2 : (-5)$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & 0 & -1 & 2 \\ 0 & 1 & 0 & 0 & 1/5 & -1/5 & 1/5 & 1/5 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & -2 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_4 \to r_4 - 2r_2} \left[\begin{array}{c|cccc} & 2 & 0 & -1 & 2 \\ I_4 & \frac{1}{5} & -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ & -1 & 0 & 1 & -2 \\ & -\frac{2}{5} & \frac{2}{5} & -\frac{2}{5} & \frac{3}{5} \end{array}\right]$$

A^{-1}

this almost look like a perfect 4rd row of I_4, I just need zero instead two and than I will have matrix I_4 in front of the line.

So, finally my inversing matrix is:

$$A^{-1} = \begin{bmatrix} 2 & 0 & -1 & 2 \\ 1/5 & -1/5 & 1/5 & 1/5 \\ -1 & 0 & 1 & -2 \\ -2/5 & 2/5 & -2/5 & 3/5 \end{bmatrix}$$