How to invert matrices using the Gaussian elimination algorithm.

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I Example: $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 5 & 1 \\ 2 & 8 & 2 \end{bmatrix}$

First of all, I need to rewrite my matrix, the follow it by a vertical line and a unit matrix of appropriete dimensions.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 1 & 5 & 1 & 0 & 1 & 0 \\ 2 & 8 & 2 & 0 & 0 & 1 \end{array}\right]$$

I would like to have "zero" here

My goal is to perform a certain number of operations that will produce a matrix: $\left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & A^{-1} \\ 0 & 0 & 1 & \end{array}\right]$, I_3

$$\xrightarrow[r_2 \to r_2 - r_3]{r_3 \to r_3 : 2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -\frac{1}{2} \\ 1 & 4 & 1 & 0 & 0 & \frac{1}{2} \end{array}\right] \xrightarrow{r_3 \to r_3 - r_1} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 5 & 2 & -1 & 0 & \frac{1}{2} \end{array}\right] \longrightarrow$$

If I get rid of this number, my first column will look exactly like I_3

I would like to have "zero" here.

$$\xrightarrow{r_3 \to r_3 - 5r_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 2 & -1 & -5 & 3 \end{array}\right] \xrightarrow{r_1 \to r_1 + r_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 1 & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 2 & -1 & -5 & 3 \end{array}\right] \xrightarrow[r_1 \to r_1 + r_3]{r_3 \to r_3 : 2} \left[\begin{array}{c|ccc} & \frac{1}{2} & -\frac{3}{2} & 1 \\ I_3 & 0 & 1 & -\frac{1}{2} \\ & -\frac{1}{2} & -\frac{5}{2} & \frac{3}{2} \end{array}\right]$$

If I get rid of this number, my second column will look like I_3.

If I get rid of this numbers, my third column will look like I_3.

Answer: $A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{3}{2} & 1 \\ 0 & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{5}{2} & \frac{3}{2} \end{bmatrix}$

II Example: $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 3 & 6 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_3 \to r_3 - r_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & -1 & 1 \end{array}\right] \xrightarrow{r_2 \to r_2 - r_1} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 1 & 3 & 0 & -1 & 1 \end{array}\right] \longrightarrow$$

I would like to have "0" here.

If I get rid of this number, my first column will look like I_3.

I would like to have "0" here.

$$\xrightarrow{r_3 \to r_3 - r_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right] \xrightarrow{r_2 \to r_2 - 2r_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & 5 & -2 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right] \xrightarrow[r_1 \to r_1 - r_3]{r_1 \to r_1 - r_2} \left[\begin{array}{c|ccc} & 3 & -3 & -3 \\ I_3 & -3 & 5 & -2 \\ & 1 & -2 & 1 \end{array}\right]$$

I would like to have "0" here.

Answer: $A^{-1} = \begin{bmatrix} 3 & -3 & -3 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{bmatrix}$