my matrix: $A = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ my goal is to perform certain number of operations that will give me matrix A^{-1}.

$\left[\begin{array}{cccc|cccc} 1 & -1 & -1 & 1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$ I rewrite matrix "upside-down" it is just easier for me then → $r_1 \leftrightarrow r_4$, $r_2 \leftrightarrow r_3$

$\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & -1 & -1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & -1 & -1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \end{array}\right]$ I want to get rid of 1's in circles → $r_2 = r_2 - r_1$, $r_3 = r_3 - r_4$, $r_4 = r_4 - r_2$

$\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & -2 & 0 & 0 & 1 & -1 \\ 0 & 0 & 2 & -2 & -1 & 1 & 0 & 0 \\ 0 & -2 & 0 & 2 & 1 & 0 & -1 & 0 \end{array}\right]$ I see that r_4 can possibly be r_2 and r_2 and r_3 should be at the bottom I do it in next 2 steps

$r_3 \leftrightarrow r_4$ → $\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -2 & 2 & 0 & 0 & 1 & -1 \\ 0 & -2 & 0 & 2 & 1 & 0 & -1 & 0 \\ 0 & 0 & 2 & -2 & -1 & 1 & 0 & 0 \end{array}\right]$ $r_2 \leftrightarrow r_3$ → $\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & 2 & 1 & 0 & -1 & 0 \\ 0 & 0 & -2 & -2 & 0 & 0 & 1 & -1 \\ 0 & 0 & 2 & -2 & -1 & 1 & 0 & 0 \end{array}\right]$

I want to have 1 instead of -2
I would like to transform r_3 and r_4 into a form more similar to that from unit matrix

$r_2 \cdot (-1/2)$, $r_3 = r_3 - r_4$, $r_4 = r_3 + r_4$ → $\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & -4 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & -4 & -1 & 1 & 1 & -1 \end{array}\right]$

I have almost perfect r_3 and r_4 → $r_3 : (-4)$, $r_4 : (-4)$

$\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & -1/4 & 1/4 & -1/4 & 1/4 \\ 0 & 0 & 0 & 1 & 1/4 & -1/4 & -1/4 & 1/4 \end{array}\right]$

I want to have perfect r_2
I see that using perfect r_3 and r_4 I can simplify a bit my r_1

$r_1 = r_1 - r_3 - r_4$, $r_2 = r_2 + r_4$ → $\left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & -1/4 & -1/4 & 1/4 & 1/4 \\ 0 & 0 & 1 & 0 & -1/4 & 1/4 & -1/4 & 1/4 \\ 0 & 0 & 0 & 1 & 1/4 & -1/4 & -1/4 & 1/4 \end{array}\right]$

the only thing that spoils my unit matrix is 1 in the circle. I can get rid of it using r_2 → $r_1 = r_1 - r_2$

$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 1 & 0 & 0 & -1/4 & -1/4 & 1/4 & 1/4 \\ 0 & 0 & 1 & 0 & -1/4 & 1/4 & -1/4 & 1/4 \\ 0 & 0 & 0 & 1 & 1/4 & -1/4 & -1/4 & 1/4 \end{array}\right]$

A^{-1}

it seems to be complicated, but can be written as: $\frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$

AK.
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