How to invert matrices using the Gaussian elimination algorithm.

Example: $A = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & 1 \\ -1 & 2 & -1 \end{bmatrix}$

First of all, I need to rewrite my matrix, then follow it by a vertical line and a unit matrix of appropriate dimensions.

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ -1 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

My goal is to perform a certain number of operations that will produce a matrix:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & A^{-1} \\ 0 & 0 & 1 & \end{array}\right]$$

(I_3)

It is very comfortable to start with first column. As, we can see the first column looks almost perfect, I only have to get rid of -1. The best way to get rid of -1 is adding 1st row to the third one.

$r_3 \to r_3 + r_1$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

$r_1 \to r_1 + 2r_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

$r_1 \to r_1 - 4r_3$

The first column looks perfect, so let's move to the second one. If I get rid of -2 the second column will look exactly like in I_3. I can divide first row by -2. (but this will produce fractions!) or I can add 2 times 2 row.

Here, I can not divide first row by four because "1" will vanish, so I can substract 4 times third row from the first row.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & -4 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

$r_2 \to r_2 - r_3$

$$\left[\begin{array}{ccc|ccc} & & & -3 & 2 & -4 \\ & I_3 & & -1 & 1 & -1 \\ & & & 1 & 0 & 1 \end{array}\right]$$

A^{-1}

If I get rid off this number I will have matrix I_3 in front of the line

Author: Natalia Korcz
(EPM, II semester)