

How to invert matrices using the Gaussian elimination algorithm

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(EPM I semester)

Example:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

First of all, I need to rewrite my matrix, then follow it by a vertical line a unit matrix of appropriate dimensions.

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 1 \end{array} \right]$$

My goal is to perform a certain number of operations that will produce a matrix

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & & & & \\ 0 & 1 & 0 & 0 & & & & \\ 0 & 0 & 1 & 0 & & & & \\ 0 & 0 & 0 & 1 & & & & \end{array} \right] A^{-1}$$

It's very comfortable when I take 2 row, 3, 4 row and out of 1 row.

$$\begin{array}{l} r_2 \rightarrow r_1 \\ r_3 \rightarrow r_1 \\ r_4 \rightarrow r_1 \end{array} \rightarrow \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & -2 & -1 & 1 & 0 & 0 \\ 0 & -2 & 0 & -2 & -1 & 0 & 1 & 0 \\ 0 & -2 & -2 & 0 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} r_2/(-2) \\ r_3/(-2) \\ r_4/(-2) \end{array} \rightarrow \left[\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1/2 & -1/2 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1/2 & 0 & -1/2 & 0 \\ 0 & 1 & 1 & 0 & 1/2 & 0 & 0 & -1/2 \end{array} \right]$$

I divide by (-2)

I divide by (-2), because I would like to have 1, not to have (-2).

$$\begin{array}{l} r_1 \rightarrow r_4 \\ r_2 \leftrightarrow r_3 \\ r_4 \rightarrow r_3 \end{array} \rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1/2 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 1 & 1/2 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & 1 & 1/2 & -1/2 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 1/2 & -1/2 \end{array} \right]$$

$$\begin{array}{l} r_4 \rightarrow r_3 \end{array} \rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1/2 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 1 & 1/2 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & 1 & 1/2 & -1/2 & 0 & 0 \\ 0 & 0 & 0 & -2 & -1/2 & 1/2 & 1/2 & -1/2 \end{array} \right]$$

I will get out of these number

I must divide by (-2) (again)

$$\begin{array}{l} r_4/(-2) \end{array} \rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & 1/2 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 1 & 1/2 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & 1 & 1/2 & -1/2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1/4 & -1/4 & -1/4 & 1/4 \end{array} \right]$$

$$\begin{array}{l} r_1 \rightarrow r_4 \\ r_2 \rightarrow r_4 \\ r_3 \rightarrow r_4 \end{array} \rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 1 & 0 & 0 & 1/2 & 1/2 & -1/4 & -1/4 \\ 0 & 0 & 1 & 0 & 1/4 & -1/4 & 1/4 & -1/4 \\ 0 & 0 & 0 & 1 & 1/4 & -1/4 & -1/4 & 1/4 \end{array} \right]$$

this looks exactly like 4th row in I_n

I_n A^{-1}