11. 05. 11.

How to invert matrices using the Gaussan elimination algorithm.

Example: $A = \begin{bmatrix} 1&1&0&0&0 \\ 0&1&1&0&0 \\ 0&0&1&1&0 \\ 0&0&0&1&1 \\ 9&0&0&0&1 \end{bmatrix}$ $J_5 = \begin{bmatrix} 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \end{bmatrix}$

Firstly, I have to rewrite my matrix, then follow it by a vertical line and unit matrix of appropriate dimensions.

$$\left[\begin{array}{ccccc|ccccc} 1&1&0&0&0&1&0&0&0&0 \\ 0&1&1&0&0&0&1&0&0&0 \\ 0&0&1&1&0&0&0&1&0&0 \\ 0&0&0&1&1&0&0&0&1&0 \\ 9&0&0&0&1&0&0&0&0&1 \end{array}\right]$$

my goal is to perform operations that will produce a matrix:

$$\left[\begin{array}{c|c} J_5 & A^{-1} \end{array}\right]$$

→ it's very comfortable to have "1" in the left corner.

→ I want to have "0" here, then this column will look like in J_5

$r_5 = r_5 - 9r_1$

$$\left[\begin{array}{ccccc|ccccc} 1&1&0&0&0&1&0&0&0&0 \\ 0&1&1&0&0&0&1&0&0&0 \\ 0&0&1&1&0&0&0&1&0&0 \\ 0&0&0&1&1&0&0&0&1&0 \\ 0&-9&0&0&1&-9&0&0&0&1 \end{array}\right]$$

$r_1 = r_1 - r_2$, $r_5 = r_5 + 9r_2$

$$\left[\begin{array}{ccccc|ccccc} 1&0&0&0&0&1&-1&0&0&0 \\ 0&1&1&0&0&0&1&0&0&0 \\ 0&0&1&1&0&0&0&1&0&0 \\ 0&0&0&1&1&0&0&0&1&0 \\ 0&0&9&0&1&-9&9&0&0&1 \end{array}\right]$$

$r_1 = r_1 + r_3$, $r_2 = r_2 - r_3$, $r_5 = r_5 - 9r_3$

→ I want to have "0" here

→ I want to get rid of these numbers.

$$\left[\begin{array}{ccccc|ccccc} 1&0&0&0&0&1&-1&1&0&0 \\ 0&1&0&0&0&0&1&-1&0&0 \\ 0&0&1&0&0&0&0&1&0&0 \\ 0&0&0&1&1&0&0&0&1&0 \\ 0&0&0&0&1&-9&9&-9&0&1 \end{array}\right]$$

$r_1 = r_1 - r_4$, $r_2 = r_2 + r_4$, $r_3 = r_3 - r_4$, $r_5 = r_5 + 9r_4$

$$\left[\begin{array}{ccccc|ccccc} 1&0&0&0&-1&1&-1&1&-1&0 \\ 0&1&0&0&1&0&1&-1&1&0 \\ 0&0&1&0&-1&0&0&1&-1&0 \\ 0&0&0&1&1&0&0&0&1&0 \\ 0&0&0&0&10&-9&9&-9&9&1 \end{array}\right]$$

$r_5 = r_5 : 10$

→ I want to have zeros here

→ I want to have "1" here

$$\left[\begin{array}{ccccc|ccccc} 1&0&0&0&-1&1&-1&1&-1&0 \\ 0&1&0&0&1&0&1&-1&1&0 \\ 0&0&1&0&-1&0&0&1&-1&0 \\ 0&0&0&1&1&0&0&0&1&0 \\ 0&0&0&0&1&-9/10&9/10&-9/10&9/10&1/10 \end{array}\right]$$

$r_1 = r_1 + r_5$, $r_2 = r_2 - r_5$, $r_3 = r_3 + r_5$, $r_4 = r_4 - r_5$

$$\left[\begin{array}{ccccc|ccccc} 1&0&0&0&0&1/10&-1/10&1/10&-1/10&1/10 \\ 0&1&0&0&0&9/10&1/10&-1/10&1/10&-1/10 \\ 0&0&1&0&0&-9/10&9/10&1/10&-1/10&1/10 \\ 0&0&0&1&0&9/10&-9/10&9/10&1/10&-1/10 \\ 0&0&0&0&1&-9/10&9/10&-9/10&9/10&1/10 \end{array}\right]$$

→ I want to have zeros in that places, to obtain J_5 matrix.

$$\left[\begin{array}{c|c} J_5 & A^{-1} \end{array}\right]$$

Obtained matrix is the inverse of the matrix A.

This is A^{-1}

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