Emilia Rast EPM

# How to invert matrices using the Gaussian elimination algorithm.

Example $A = \begin{bmatrix} 1210 \\ 0142 \\ 1320 \\ 0321 \end{bmatrix}$

First of all, I need to rewrite my matrix, then follow it by a vertical line and a unit matrix of appropriate dimensions

My goal is to perform a certain number of operations that will produce a matrix:

$$\left[\begin{array}{c|c} 1000 \\ 0100 \\ 0010 \\ 0001 \end{array} \right. \left. A^{-1} \right]$$

$I_4$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 2 & 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\substack{r_3 \to r_3 - r_1 \\ r_2 \to r_2 - 2r_4}} \left[\begin{array}{cccc|cccc} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -5 & 0 & 0 & 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\substack{r_2 : (-5) \\ r_4 \to r_4 - 2r_3}} \left[\begin{array}{cccc|cccc} 1 & 2 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{5} & 0 & \frac{2}{5} & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 2 & 0 & -2 & 1 \end{array}\right]$$

I would like to have 0 instead of 1 because then my first column will look like in $I_4$

I would like to have 1 here, instead of -5

I would like to have two zeroes. One instead of 2 and another instead of 1

$$\xrightarrow{\substack{r_4 \to r_4 - r_3 \\ r_1 \to r_1 - 2r_2}} \left[\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & \frac{7}{5} & 0 & -\frac{4}{5} & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{5} & 0 & \frac{2}{5} & 0 \\ 0 & 0 & 1 & 0 & -\frac{4}{5} & 0 & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 1 & \frac{11}{5} & 0 & -\frac{12}{5} & 1 \end{array}\right] \xrightarrow{\substack{r_4 \to r_4 + r_3 \\ r_1 \to r_1 - r_3}} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{11}{5} & 0 & -\frac{16}{5} & 1 \\ 0 & 1 & 0 & 0 & -\frac{1}{5} & 0 & \frac{2}{5} & 0 \\ 0 & 0 & 1 & 0 & -\frac{4}{5} & 0 & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 1 & \frac{11}{5} & 0 & -\frac{12}{5} & 1 \end{array}\right] \xrightarrow{r_1(-1)} \left[\begin{array}{cccc|cccc} -1 & 0 & 0 & 0 & -\frac{11}{5} & 0 & \frac{16}{5} & -1 \\ 0 & 1 & 0 & 0 & -\frac{1}{5} & 0 & \frac{2}{5} & 0 \\ 0 & 0 & 1 & 0 & -\frac{4}{5} & 0 & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 1 & \frac{11}{5} & 0 & -\frac{12}{5} & 1 \end{array}\right]$$

It will be convenient to have in 4th row 1 instead of 0 and 0 instead of -1

I would like to have in first row 1 instead of -1. That's why I'm gonna multiple first row by (-1)

I would like to have 0 instead of 1 in first and third row

$$\xrightarrow{\substack{r_3 \to r_3 - r_2 \\ r_1 \to r_1 - r_2}} \left[\begin{array}{cccc|cccc} -1 & 0 & 0 & 0 & -2 & -\frac{1}{5} & 1 & \frac{2}{5} \\ 0 & 1 & 0 & 0 & 0 & -\frac{1}{5} & 0 & \frac{2}{5} \\ 0 & 0 & 1 & 0 & -1 & -\frac{1}{5} & 1 & -\frac{2}{5} \\ 0 & 1 & 0 & 1 & 2 & 0 & -2 & 1 \end{array}\right] \xrightarrow{\substack{r_1(-1) \\ r_4 \to r_4 - r_2}} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & \frac{1}{5} & -1 & -\frac{2}{5} \\ 0 & 1 & 0 & 0 & 0 & -\frac{1}{5} & 0 & \frac{2}{5} \\ 0 & 0 & 1 & 0 & -1 & \frac{1}{5} & 1 & -\frac{2}{5} \\ 0 & 0 & 0 & 1 & 2 & -\frac{1}{5} & -2 & \frac{3}{5} \end{array}\right]$$

I would like to have 0 instead of 1, so I will achive last row the same as in $I_4$

$$A^{-1}$$