EXTREMES OF 2-VARIABLE FUNCTIONS

$$f(x,y) = xy^2(12-x-y)$$
$$f(x,y) = 12xy^2 - x^2y^2 - xy^3$$

I PARTIAL DERIVATIVES:

$$\frac{\partial f}{\partial x} = 12y^2 - 2xy^2 - y^3 \qquad \frac{\partial^2 f}{\partial x^2} = -2y^2$$

$$\frac{\partial f}{\partial y} = 24xy - 2yx^2 - 3y^2x \qquad \frac{\partial^2 f}{\partial y^2} = 24x - 2x^2 - 6yx$$

$$\frac{\partial^2 f}{\partial x \partial y} = 24y - 4yx - 3y^2$$

II CRITICAL (STATIONARY) POINTS

$$\begin{cases} 12y^2 - 2xy^2 - y^3 = 0 \\ 24xy - 2yx^2 \cdot 3y^2x = 0 \end{cases} \Longrightarrow \quad y^2(12-2x-y) = 0$$

$$y = 0 \quad \vee \quad y = 12-2x$$

$y = 12-2x$:

$$24x(12-2x) - 2(12-2x)x^2 - 3(12\cdot 2x)^2x = 0$$
$$288x - 48x^2 - 24x^2 + 4x^3 - 3x(144-48x+4x^2) = 0$$
$$288x - 48x^2 - 24x^2 + 4x^3 - 432x + 144x^2 - 12x^3 = 0$$
$$-8x^3 + 72x^2 - 144x = 0 \quad /:8$$
$$-x^3 + 9x^2 - 18x = 0$$
$$x(-x^2 + 9x - 18 = 0) = 0$$

$$x = 0 \quad \vee \quad -x^2 + 9x - 18 = 0$$
$$\Delta = 9^2 - 4\cdot(18)\cdot(-1) = 9 \quad \Rightarrow \sqrt{\Delta} = 3$$
$$x_1 = \frac{-9-3}{-2} = 6 \qquad x_2 = \frac{-9+3}{-2} = 3$$

$$P_1 = (0,0) \qquad P_2 = (6,0) \qquad P_3 = (3,6)$$

III DETERMINANTS

$$D_f = \begin{vmatrix} -2y^2 & 24y-4xy-3y^2 \\ 24y-4xy-3y^2 & 24x-2x^2-6yx \end{vmatrix} =$$

$$= (-2y^2)(24x-2x^2-6yx) - \left((24y-4xy-3y^2)(24y-4xy-3y^2)\right) =$$

$$= -9y^4 - 12xy^3 + 144y^3 - 12x^2y^2 + 144xy^2 - 576y^2$$

$$D_f(0,0) = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$D_f(6,0) = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$D_f(3,6) = \begin{vmatrix} -72 & -36 \\ -36 & -54 \end{vmatrix} = 3888 - 1296 = 2592 > 0$$

$$(-2y^2)\Big|_{(3,6)} = -72 < 0 \qquad \text{maximum}$$

IV ANSWER

$f(3,6) = 12 \cdot 3 \cdot 6^2 - 3^2 6^2 - 3 \cdot 6^3 = \underline{324}$ is a maximum

As the value of determinants in points $P_1 = (0,0)$ and $P_2 = (6,0)$ are equal to zero we can not state whether they are minimum values or saddle points.

KAMILA STANKIEWICZ

Here, I present two graphs of f(x,y) in a close surrounding of points (0,0) and (3,6). The surrounding of (0,0) here is [-0.1, 0.1] x [-0.1, 0.1] and the surrounding of (3,6) is [2.9, 3.1] x [5.9, 6.1].