

EXTREMES OF 2-VARIABLE FUNCTIONS

$$f(x,y) = xy^2(12-x-y)$$
$$f(x,y) = 12xy^2 - x^2y^2 - xy^3$$

I PARTIAL DERIVATIVES:

$$\frac{\partial f}{\partial x} = 12y^2 - 2xy^2 - y^3$$

$$\frac{\partial^2 f}{\partial x^2} = -2y^2$$

$$\frac{\partial f}{\partial y} = 24xy - 2yx^2 - 3y^2x$$

$$\frac{\partial^2 f}{\partial y^2} = 24x - 2x^2 - 6yx$$

$$\frac{\partial^2 f}{\partial x \partial y} = 24y - 4yx - 3y^2$$

II CRITICAL (STATIONARY) POINTS

$$\begin{cases} 12y^2 - 2xy^2 - y^3 = 0 \\ 24xy - 2yx^2 - 3y^2x = 0 \end{cases} \Rightarrow \begin{cases} y^2(12 - 2x - y) = 0 \\ y = 0 \vee y = 12 - 2x \end{cases}$$

$$y = 12 - 2x:$$

$$24x(12-2x) - 2(12-2x)x^2 - 3(12-2x)^2x = 0$$

$$288x - 48x^2 - 24x^2 + 4x^3 - 3x(144 - 48x + 4x^2) = 0$$

$$288x - 48x^2 - 24x^2 + 4x^3 - 432x + 144x^2 - 12x^3 = 0$$

$$-8x^3 + 72x^2 - 144x = 0 \quad /: 8$$

$$-x^3 + 9x^2 - 18x = 0$$

$$x(-x^2 + 9x - 18 = 0) = 0$$

$$x = 0 \quad \vee \quad -x^2 + 9x - 18 = 0$$

$$\Delta = 9^2 - 4 \cdot (18) \cdot (-1) = 9 \Rightarrow \sqrt{\Delta} = 3$$

$$x_1 = \frac{-9-3}{-2} = 6 \quad x_2 = \frac{-9+3}{-2} = 3$$

$$P_1 = (0, 0)$$

$$P_2 = (6, 0)$$

$$P_3 = (3, 6)$$

III DETERMINANTS

$$Df = \begin{vmatrix} -2y^2 & 24y - 4xy - 3y^2 \\ 24y - 4xy - 3y^2 & 24x - 2x^2 - 6y^2 \end{vmatrix} =$$
$$= (-2y^2)(24x - 2x^2 - 6y^2) - ((24y - 4xy - 3y^2)(24y - 4xy - 3y^2)) =$$
$$= -9y^4 - 12xy^3 + 144y^3 - 12x^2y^2 + 144xy^2 - 576y^2$$

$$Df(0,0) = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$Df(6,0) = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$Df(3,6) = \begin{vmatrix} -72 & -36 \\ -36 & -54 \end{vmatrix} = 3888 - 1296 = 2592 > 0$$

$$\left. (-2y^2) \right|_{(3,6)} = -72 < 0 \quad \text{maximum}$$

IV ANSWER

$$f(3,6) = 2 \cdot 3 \cdot 6^2 - 3^2 \cdot 6^2 - 3 \cdot 6^3 = \underline{324} \quad \text{is a maximum}$$

As the value of determinants in points $P_1(0,0)$ and $P_2(6,0)$ are equal to zero we can not state whether they are minimum values or saddle points.

Here, I present two graphs of $f(x,y)$ in a close surrounding of points $(0,0)$ and $(3,6)$. The surrounding of $(0,0)$ here is $[-0.1, 0.1] \times [-0.1, 0.1]$ and the surrounding of $(3,6)$ is $[2.9, 3.1] \times [5.9, 6.1]$.



plot $12xy^2 - x^2y^2 - xy^3$, $x=-0.1..0.1$, $y=-0.1..0.1$

Input interpretation:

plot

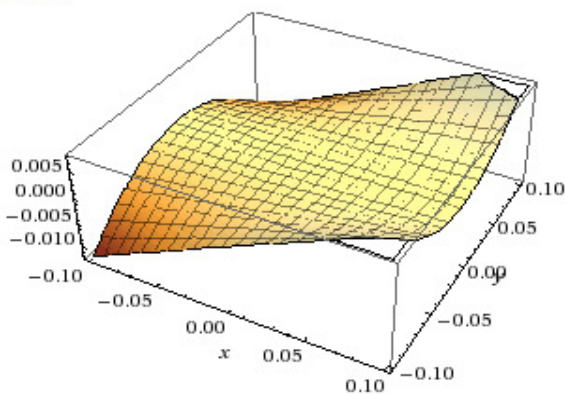
$$12xy^2 - x^2y^2 - xy^3$$

$x = -0.1$ to 0.1

$y = -0.1$ to 0.1

3D plot:

Show contour lines



plot $12xy^2 - x^2y^2 - xy^3$, $x=2.9..3.1$, $y=5.9..6.1$

Input interpretation:

plot

$$12xy^2 - x^2y^2 - xy^3$$

$x = 2.9$ to 3.1

$y = 5.9$ to 6.1

3D plot:

Show contour lines

