Extremes of 2-variable functions

Alicja Szpejnowska
EPM 2, gr. 3

$F(x,y) = x^3 + 8y^3 - 6xy + 2$

I. Partial derivatives

$\frac{\partial f}{\partial x} = 3x^2 - 6y$ $\quad \frac{\partial^2 f}{\partial x^2} = 6x$ $\quad \frac{\partial f}{\partial x \partial y} = -6$

$\frac{\partial f}{\partial y} = 24y^2 - 6x$ $\quad \frac{\partial^2 f}{\partial y^2} = 48y$

II. Critical points

$\begin{cases} 3x^2 - 6y = 0 \\ 24y^2 - 6x = 0 \end{cases} \Rightarrow 3x^2 = 6y \,/ : 6$

$\frac{x^2}{2} = y$

$24\left(\frac{x^2}{2}\right)^2 - 6x = 0 \,/ : 6$

$4\left(\frac{x^4}{4}\right) - x = 0$

$x^4 = x$

$\underline{x = 1}$

$\Rightarrow \begin{cases} 3x^2 - 6y = 0 \\ x = 1 \end{cases}$

$3 - 6y = 0$

$-6y = -3 \,/ : (-6)$

$\underline{y = \frac{1}{2}}$

We do not have a constant value in the formulas, so the first critical point will be: $P_1 = (0,0)$

$P_2 = (1, \frac{1}{2})$

III. Determinants

$Df = \begin{vmatrix} 6x & -6 \\ -6 & 48y \end{vmatrix}$

$Df(0,0) = \begin{vmatrix} 0 & -6 \\ -6 & 0 \end{vmatrix} = -36 < 0 \Rightarrow$ SADDLE POINT

$Df(1, \frac{1}{2}) = \begin{vmatrix} 6 & -6 \\ -6 & 24 \end{vmatrix} = 144 - 36 = 108 > 0 \Rightarrow$ minimum

$(6x)\Big|_{(1,\frac{1}{2})} = 6 > 0$

IV. Answer

$f(1, \frac{1}{2}) = 1^3 + 8 \cdot (\frac{1}{2})^3 - 6 \cdot 1 \cdot \frac{1}{2} + 2 = 1 + 8 \cdot \frac{1}{8} - 3 + 2 = 1 + 1 - 3 + 2 = \underline{1}$

$f(1, \frac{1}{2}) = 1$ is a minimum

$f(0,0) = 0$ is a saddle point

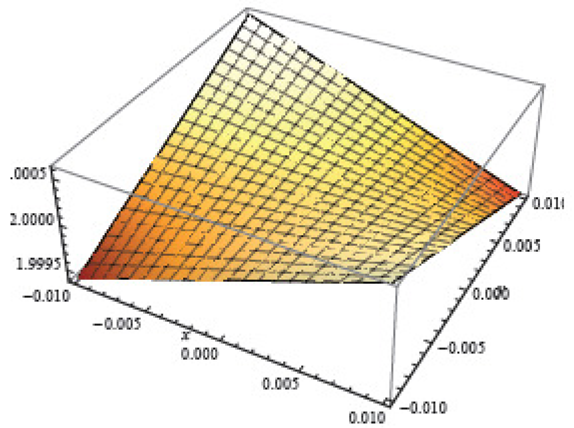
$P = (0,0,0)$

plot x^3+8*y^3−6*x*y+2 from x=−0.01 to 0.01, from y=−0.01 to 0.01

Input interpretation:

plot	$x^3+8y^3-6xy+2$	$x = -0.01$ to 0.01
		$y = -0.01$ to 0.01

3D plot:



plot x^3+8*y^3−6*x*y+2 from x=0.99 to 1.01, from y=0.49 to 0.51

Input interpretation:

plot	$x^3+8y^3-6xy+2$	$x = 0.99$ to 1.01
		$y = 0.49$ to 0.51

3D plot: