Extremes of 2-variable functions

$$f(x,y) = 2x^2 + y^2 + 3xy - 2x - y + 1$$

I Partial derivatives:

$$\frac{\partial f}{\partial x} = 4x + 3y - 2 \qquad \frac{\partial^2 f}{\partial x^2} = 4$$

$$\frac{\partial f}{\partial y} = 2y + 3x - 1 \qquad \frac{\partial^2 f}{\partial y^2} = 2$$

$$\frac{\partial^2 f}{\partial x \partial y} = (4x + 3y - 2)'_y = 3$$

II Critical points

$$\begin{cases} 4x + 3y - 2 = 0 \\ 2y + 3x - 1 = 0 \end{cases} \Rightarrow \begin{array}{l} 4x + 2 - 3y = 0 \mid : 4 \\ x = \frac{1}{2} - \frac{3}{4}y \end{array}$$

$$\begin{cases} x = \frac{1}{2} - \frac{3}{4}y \\ 2y + 3\left(\frac{1}{2} - \frac{3}{4}y\right) - 1 = 0 \end{cases} \Rightarrow 2y + \frac{3}{2} - \frac{9}{4}y - 1 = 0$$

$$-\frac{1}{4}y + \frac{1}{2} = 0 \;,\; -\frac{1}{4}y = -\frac{1}{2} \quad \mid \cdot (-4)$$

$y = 2$, hence:

$$x = \frac{1}{2} - \frac{3}{4} \cdot 2$$

$$x = -1$$

$$P(-1, 2)$$

III Determinant

$$D_f = \begin{vmatrix} \overset{\frac{\partial^2 f}{\partial x^2}}{4} & \overset{\frac{\partial^2 f}{\partial x \partial y}}{3} \\ \underset{\partial^2 f}{3} & \underset{\frac{\partial^2 f}{\partial y^2}}{2} \end{vmatrix} = 8 - 9 = -1$$

Thus: $D_f(-1, 2) = -1 < 0$, saddle point

IV

Answer

$$f(-1,2) = 2\cdot(-1)^2 + 2^2 + 3\cdot(-1)\cdot 2 - 2\cdot(-1) - 2 + 1 = 2+4-6+2-2+1 = 1$$

Saddle point: $P(-1, 2, 1)$

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WolframAlpha computational knowledge engine

plot 2*x^2+y^2+3*x*y−2*x−y+1, x=−1.1..−0.9, y=1.9..2.1

Input interpretation:

plot	$2x^2+y^2+3xy-2x-y+1$	$x = -1.1$ to -0.9
		$y = 1.9$ to 2.1

3D plot: