Extremes of two variable function

$$f(x,y) = x^3 + y^3 - 9xy + 27$$

I Partial derivatives

$$\frac{\partial f}{\partial x} = 3x^2 - 9y \qquad \frac{\partial^2 f}{\partial x^2} = 6x \qquad \frac{\partial^2 f}{\partial x \partial y} = (3x^2 - 9y)'_y = -9$$

$$\frac{\partial f}{\partial y} = 3y^2 - 9x \qquad \frac{\partial^2 f}{\partial y^2} = 6y$$

II Critical stationary points

$$3x^2 - 9y = 0 \Rightarrow 3x^2 = 9y \Rightarrow x^2 = 3y \Rightarrow y = \frac{x^2}{3}$$

$$3y^2 - 9x = 0 \Rightarrow 3 \cdot \left(\frac{x^2}{3}\right)^2 - 9x = 0$$

$$\cancel{3}^1 \cdot \frac{x^4}{\cancel{9}_3} - 9x = 0 \quad / \cdot 3$$

$$x^4 - 27x = 0$$

$$x(x^3 - 27) = 0$$

$$\begin{matrix} x = 0 \\ y = 0 \end{matrix} \quad \cup \quad \begin{matrix} x = 3 \\ y = 3 \end{matrix}$$

$$P_1 = (0,0) \quad P_2 = (3,3)$$

III Determinants

$$D_f = \begin{vmatrix} 6x & -9 \\ -9 & 6y \end{vmatrix}$$

$$D_{f(0,0)} = \begin{vmatrix} 0 & -9 \\ -9 & 0 \end{vmatrix} = -81 < 0, \ (6x)\big|_{(0,0)} = 0$$

SADDLE POINT

$$D_{f(3,3)} = \begin{vmatrix} 18 & -9 \\ -9 & 18 \end{vmatrix} = 324 - 81 = 243 > 0, \ (6x)\big|_{(3,3)} = 18 > 0$$

MINIMUM

IV Answer

$f(0,0) = 27$ Saddle point

$f(3,3)$ is minimum

$= 27 + 27 - 54 + 27 = 81 - 54 = 27$

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I present two graphs of f(x,y)in close surrounding of points (0,0) and (3,3).

The surrounding of (0,0)here is (-0,1.0,1)x(-0,1.0,1)and surrounding of (3,3) is (2,9.3,1)x(2,9.3,1).

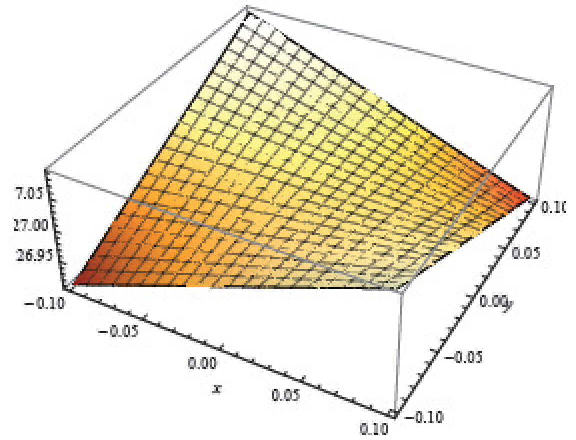
WolframAlpha computational knowledge engine

plot x^3+y^3–9xy+27, x=–0.1..0.1, y=–0.1..0.1

Input interpretation:

plot	$x^3+y^3-9\,x\,y+27$	$x=-0.1$ to 0.1
		$y=-0.1$ to 0.1

3D plot:



WolframAlpha computational knowledge engine

plot x^3+y^3–9xy+27, x=2.9..3.1, y=2.9..3.1

Input interpretation:

plot	$x^3+y^3-9\,x\,y+27$	$x=2.9$ to 3.1
		$y=2.9$ to 3.1

3D plot: