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Find extreme values of the function

j) $f(x,y) = \frac{1}{5x} + \frac{5}{y} - xy$

First we have to calculate all derivatives

$\frac{\partial f}{\partial x} = -\frac{1}{5}x^{-2} - y$ $\quad \frac{\partial^2 f}{\partial x^2} = -\frac{1}{5}(-2)x^{-3} = \frac{2}{5}x^{-3}$

$\frac{\partial f}{\partial y} = -5y^{-2} - x$ $\quad \frac{\partial^2 f}{\partial y^2} = -5(-2)y^{-3} = 10y^{-3}$ $\quad$ and $\frac{\partial^2 f}{\partial x \partial y} = -1$

Now let's find critical points

$\begin{cases} \frac{\partial f}{\partial x} = 0 \\ \frac{\partial f}{\partial y} = 0 \end{cases}$ $\quad \begin{cases} -\frac{1}{5}x^{-2} - y = 0 \\ -5y^{-2} - x = 0 \end{cases}$ $\rightarrow x = -5y^{-2}$

$-\frac{1}{5}(-5y^{-2})^{-2} - y = 0$

$-\frac{1}{5}\left((-5)^{-2}\right)y^4 - y = 0$

$-\frac{1}{5}\left(\frac{1}{25}\right)y^4 - y = 0$

$-\frac{1}{125}y^4 - y = 0$

$-y\left(\frac{1}{125}y^3 + 1\right) = 0$

so $-y = 0 \Rightarrow \boxed{y = 0}$

or $\frac{1}{125}y^3 + 1 = 0 \quad | \cdot 125$

$y^3 = -125$

$\boxed{y = -5}$

when we put y into x value it results in

$\boxed{x = 0} \vee \boxed{x = -\frac{1}{5}}$

and our point's following as $P(0,0)$ doesnt belong to the domain.

$P = \left(-\frac{1}{5}, -5\right)$

Its high time to find the value of D_f

$D_f = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix}$ so $D_f = \left(\frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y} \cdot \frac{\partial^2 f}{\partial y \partial x}\right)$

$D_f = -50 \cdot (-0{,}08) - 1 = 3$

$\frac{\partial^2 f}{\partial x^2}\left(-\frac{1}{5}, -5\right) < 0$ which means we have found MAX

ANSWER we have maximum at $P_4 = \left(-\frac{1}{5}, -5\right)$

$f_{max}(-0{,}2, -5) = \frac{1}{5 \cdot \left(-\frac{1}{5}\right)} + \frac{5}{-5} - \left(\left(-\frac{1}{5}\right)(-5)\right) = -1 + (-1) - 1 = -3$