TEST FOR FINDING MAXIMUM AND MINIMUM

MARTA NIEDŹWIECKA
EPM, sem. 3

$$f(x,y) = 2xy + \frac{1}{x} + \frac{2}{y}$$

$$D: \; x, y \in \mathbb{R} \setminus \{0\}$$

1) Partial derivatives.

$$f_x = 2y - x^{-2} \qquad f_y = 2x - 2y^{-2}$$

$$f_{xx} = 2x^{-3} \qquad f_{yy} = 4y^{-3}$$

$$f_{xy} = 0$$

2) Critical points.

$$\begin{cases} 2y - x^{-2} = 0 \\ 2x - 2y^{-2} = 0 \,/\cdot \frac{1}{2} \end{cases} \Rightarrow x - y^{-2} = 0 \Rightarrow x = y^{-2}$$

$$2y - \frac{1}{y^{-2}} = 0$$

$$2y - y^2 = 0$$

$$-y(y-2) = 0$$

$$y = 0 \quad \vee \quad y = 2$$

$y = 0 \notin D$

$$x = (2)^{-2} = \frac{1}{4}$$

$P_1 \left(\frac{1}{4}, 2\right)$

3) Maximum and minimum.

$$D_f = \begin{vmatrix} 2x^{-3} & 0 \\ 0 & 4y^{-3} \end{vmatrix} = 2x^{-3} \cdot 4y^{-3}$$

$$D_{f\left(\frac{1}{4},2\right)} = 2\left(\frac{1}{4}\right)^{-3} \cdot 4(2)^{-3} = 2 \cdot 64 \cdot 4 \cdot \frac{1}{8} = 64$$

$$f_{xx}\left(\frac{1}{4},2\right) = 2\left(\frac{1}{4}\right)^{-3} = 128 > 0$$

4) Answer

Point $P\left(\frac{1}{4}, 2\right)$ is minimum of $f(x,y) = 2xy + \frac{1}{x} + \frac{2}{y}$.

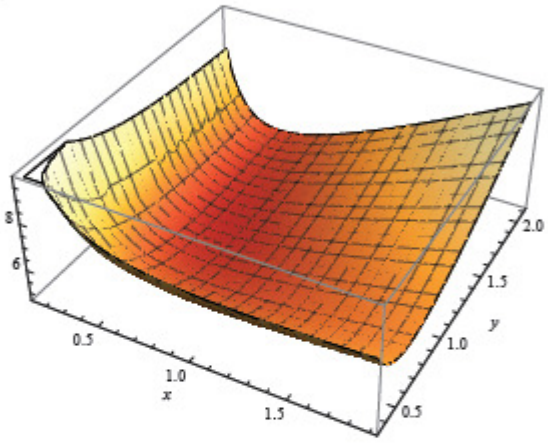
WolframAlpha computational knowledge engine

plot 2xy+1/x+2/y, x=0.15..1.9, y=0.35..2.1

Input interpretation:

plot	$2xy+\frac{1}{x}+\frac{2}{y}$	$x = 0.15$ to 1.9
		$y = 0.35$ to 2.1

3D plot:



Contour plot: