EXTREMES OF 2 VARIABLE FUNCTIONS

$f(x,y) = xy(x+y-6)$

$f(x,y) = x^2y + xy^2 - 6xy$

PARTIAL DERIVATIVES

$\frac{\partial f}{\partial x} = 2xy + y^2 - 6y \qquad \frac{\partial^2 f}{\partial x^2} = 2y$

$\frac{\partial f}{\partial y} = x^2 + 2yx - 6x \qquad \frac{\partial^2 f}{\partial y^2} = 2x$

$\frac{\partial^2 f}{\partial x \partial y} = (2xy + y^2 - 6y)'y = 2x + 2y - 6$

CRITICAL POINTS

$\begin{cases} f'x = 0 \\ f'y = 0 \end{cases} \quad \begin{cases} 2xy + y^2 - 6y = 0 \\ x^2 + 2xy - 6x = 0 \end{cases} \quad \begin{cases} y \cdot (2x + y - 6) = 0 \\ x^2 + 2xy - 6x = 0 \end{cases}$

$y = 0 \quad \vee \quad 2x + y - 6 = 0$

$y = 0$

$x^2 - 6x = 0$

$x(x-6) = 0$

$x = 0 \quad x = 6$

$P_1(0,0)$

$P_2(6,0)$

$2x + y - 6 = 0$

$y = -2x + 6$

$x^2 + 2x(-2x+6) - 6x = 0$

$-3x^2 + 6x = 0$

$x(-3x+6) = 0$

$x = 0 \quad -3x = -6 \quad x = 2$

$P_3(0,6)$

$P_4(2,2)$

DETERMINANTS

$D_f \begin{vmatrix} 2y & 2x+2y-6 \\ 2x+2y-6 & 2x \end{vmatrix} =$

$(2y \cdot 2x) - (2x+2y-6)(2x+2y-6) = 4xy - (4x^2 + 4xy - 12x + 4xy + 4y^2 - 12y - 12x - 12y + 36)$

$= 4xy - 4x^2 - 4xy + 12x - 4xy - 4y^2 + 12y + 12x + 12y - 36 =$

$-4x^2 + 24x - 4xy - 4y^2 + 24y - 36$

$D_{f(0,0)} \begin{vmatrix} 2 \cdot 0 & -6 \\ -6 & 2 \cdot 0 \end{vmatrix} = -36 < 0$ SADDLE POINT

$D_{f(6,0)} \begin{vmatrix} 0 & 6 \\ 6 & 12 \end{vmatrix} = -36 < 0$ SADDLE POINT

$D_{f(0,6)} \begin{vmatrix} 12 & 6 \\ 6 & 0 \end{vmatrix} = -36 < 0$ SADDLE POINT

$D_{f(2,2)} \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} = 16 - 4 = 12 > 0$

$\frac{\partial^2 f}{\partial x^2} = 2y$

$2y|_{(2,2)} = 4 > 0$ MINIMUM

$f(2,2) = 2^2 \cdot 2 + 8 - 24 = -8$

ANSWER

$f_{min.}(2,2) = -8$

saddle point $(0,0)$

saddle point $(6,0)$

saddle point $(0,6)$

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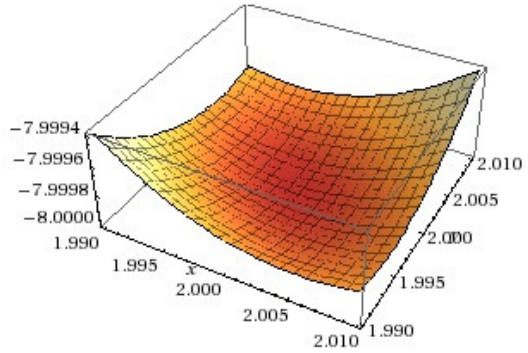
WolframAlpha computational... knowledge engine

plot x^2*y+x*y^2-6*x*y ,x=1.99..2.01 y=2.01..1.99

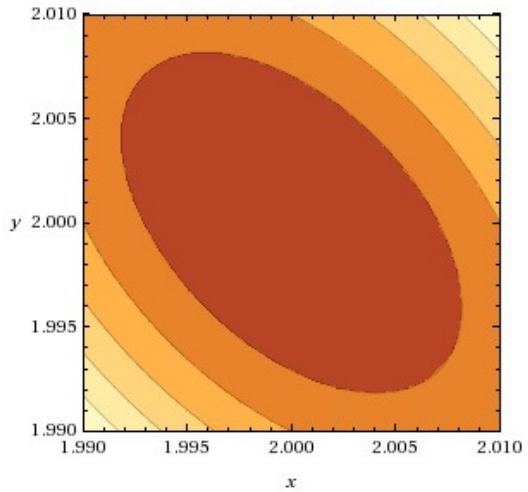
Input interpretation:

plot	$x^2\,y + x\,y^2 - 6\,x\,y$	$x = 1.99$ to 2.01
		$y = 2.01$ to 1.99

3D plot: Show contour lines



Contour plot:



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