Extrems of 2-variable functions.

$$f(x,y) = x^2y(4-x-y) = 4x^2y - x^3y - x^2y^2$$

I Partial derivatives.

$$\frac{\partial f}{\partial x} = 8xy - 3x^2y - 2xy^2$$

$$\frac{\partial f}{\partial y} = 4x^2 - x^3 - 2x^2y$$

$$\frac{\partial^2 f}{\partial x^2} = (8xy - 3x^2y - 2xy^2)'_x = 8y - 6xy - 2y^2$$

$$\frac{\partial^2 f}{\partial y^2} = (4x^2 - x^3 - 2x^2y)'_y = -2x^2$$

$$\frac{\partial^2 f}{\partial x \partial y} = (-2x^2)'_x = -4x$$

II Critical (stacionary) points.

$$\begin{cases} 8xy - 3x^2y - 2xy^2 = 0 \Rightarrow y(8x - 3x^2 - 2xy) = 0 \\ 4x^2 - x^3 - 2x^2y = 0 \end{cases}$$

$$y(8x - 3x^2 - 2xy) = 0$$

$$y = 0 \quad \vee \quad 8x - 3x^2 - 2xy = 0$$

$$-2xy = -8x + 3x^2 \quad |:(-2x)$$

$$y = 4 - \frac{3}{2}x$$

$$\begin{cases} y = 4 - \frac{3}{2}x \\ 4x^2 - x^3 - 2x^2y = 0 \end{cases}$$

$$4x^2 - x^3 - 2x^2 \cdot \left(4 - \frac{3}{2}x\right) = 0$$

$$4x^2 - x^3 - 8x^2 + 3x^3 = 0$$

$$-4x^2 + 2x^3 = 0$$

$$x^2(-4 + 2x) = 0$$

$$x = 0 \quad \vee \quad x = 2$$

$$y = 4 - \frac{3}{2}x$$

$$y = 4 - \frac{3}{2} \cdot 0 = 4 \quad \vee \quad y = 4 - \frac{3}{2} \cdot 2 = 4 - 3 = 1$$

$P_1(0,4)$ $P_2(2,1)$

$$\begin{cases} y = 0 \\ 4x^2 - x^3 - 2x^2y = 0 \Rightarrow -2x^2y = -4x^2 + x^3 \quad |:(-2x^2) \end{cases}$$

$$y = 2 - \frac{x}{2}$$

$$4x^2 - x^3 = 0$$

$$x^2(4 - x) = 0$$

$$x = 0 \quad \vee \quad x = 4$$

$$y = 2 - \frac{0}{2} = 2 \quad \vee \quad y = 2 - \frac{4}{2} = 2 - 2 = 0$$

$P_3(0,0)$ $P_4(4,0)$

III Determinants.

$$D_f = \begin{vmatrix} 8y - 6xy - 2y^2 & -4x \\ -4x & -2x^2 \end{vmatrix} = (8y - 6xy - 2y^2) \cdot (-2x^2) - (-4x \cdot (-4x)) =$$

$$= -16x^2y + 12x^3y + 4x^2y^2 - 16x^2 \quad |:(-4)$$

$$= 4x^2y - 3x^3y - x^2y^2 + 4x^2$$

$$D_f(0,4) = 0$$

$$D_f(2,1) = 4 \cdot 2^2 \cdot 1 - 3 \cdot 2^3 \cdot 1 - 2^2 \cdot 1^2 + 4 \cdot 2^2 = 16 - 24 - 4 + 16 = 32 - 28 = 4 > 0$$

$$(8y - 6xy - 2y^2)|_{(2,1)} = 8 \cdot 1 - 6 \cdot 2 \cdot 1 - 2 \cdot 1^2 = 8 - 12 - 2 = -6 < 0 \quad \text{MAXIMUM}$$

$$D_f(0,0) = 0$$

$$D_f(4,0) = 4 \cdot 4^2 = 64 > 0$$

$$(8y - 6xy - 2y^2)|_{(4,0)} = 0$$

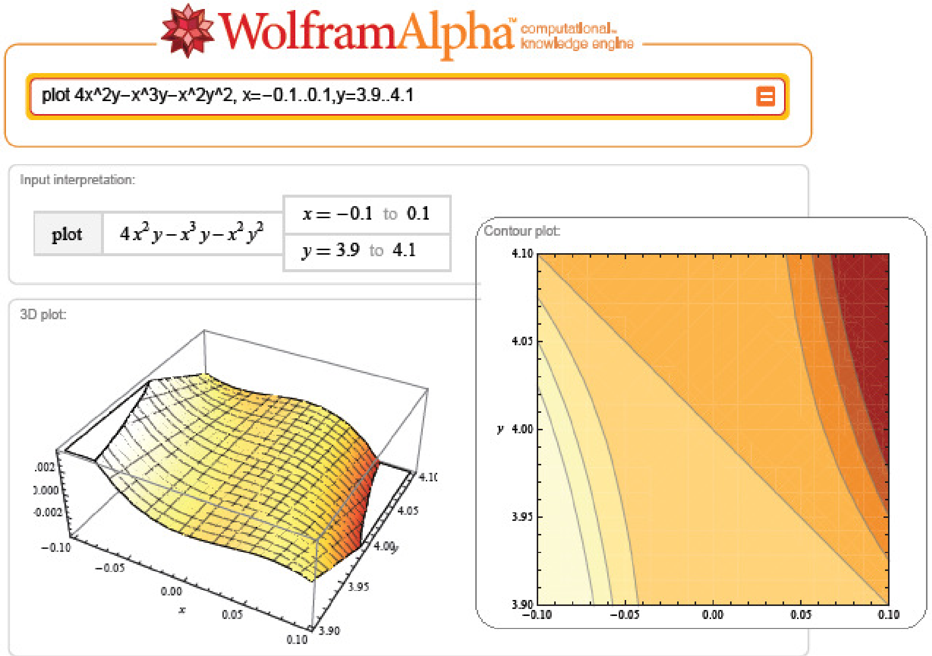
IV Answer:

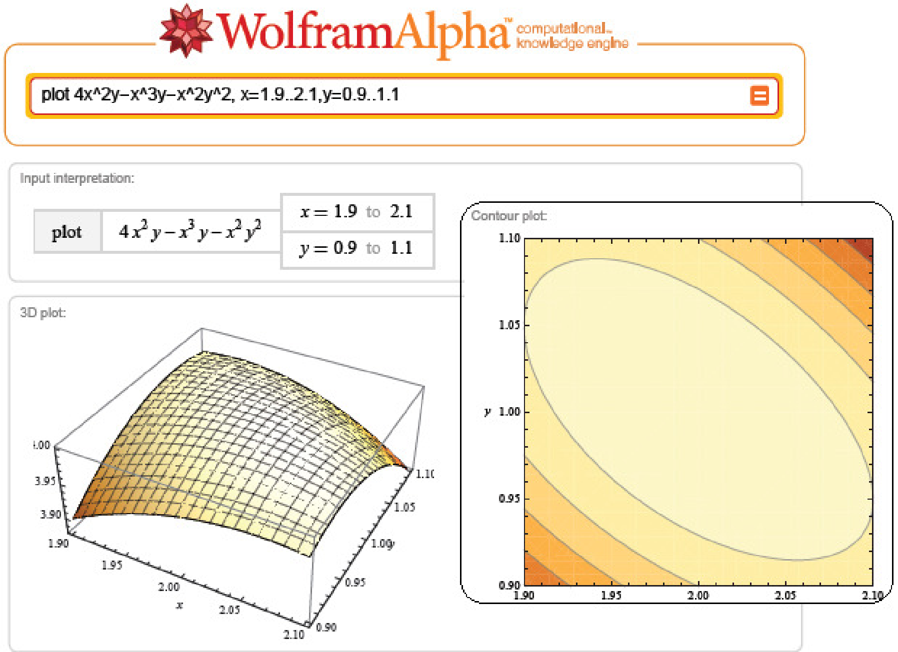
$P_2(2,1) \Rightarrow f(2,1) = 4 \cdot 2^2 \cdot 1 - 2^3 \cdot 1 - 2^2 \cdot 1^2 = 16 - 8 - 4 = 4$ is a maximum.

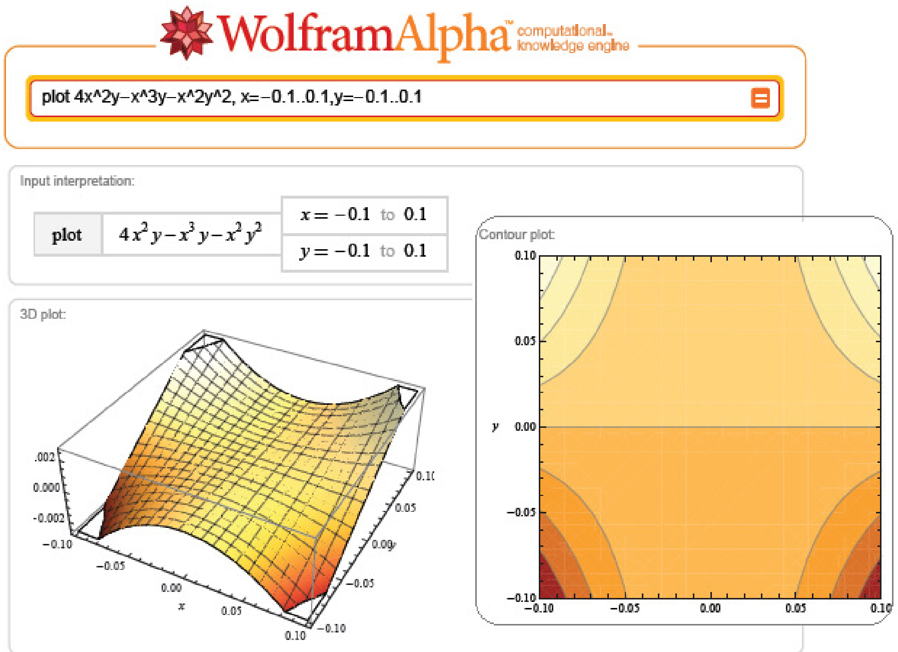
As the value of determinants is points $P_1(0,4)$ and $P_3(0,0)$ are equal to zero we can not state whether they are minimum values or saddle points.

The point $P_4(4,0)$ and it's behave, we can observe on the plot.

Aleksandra Liś EPM


WolframAlpha computational knowledge engine
plot 4x^2y–x^3y–x^2y^2, x=–0.1..0.1,y=3.9..4.1
Input interpretation:
plot
$4x^2y - x^3y - x^2y^2$
$x = -0.1$ to 0.1
$y = 3.9$ to 4.1
3D plot:
Contour plot:


WolframAlpha computational knowledge engine
plot 4x^2y–x^3y–x^2y^2, x=1.9..2.1,y=0.9..1.1
Input interpretation:
plot
$4x^2y - x^3y - x^2y^2$
$x = 1.9$ to 2.1
$y = 0.9$ to 1.1
3D plot:
Contour plot: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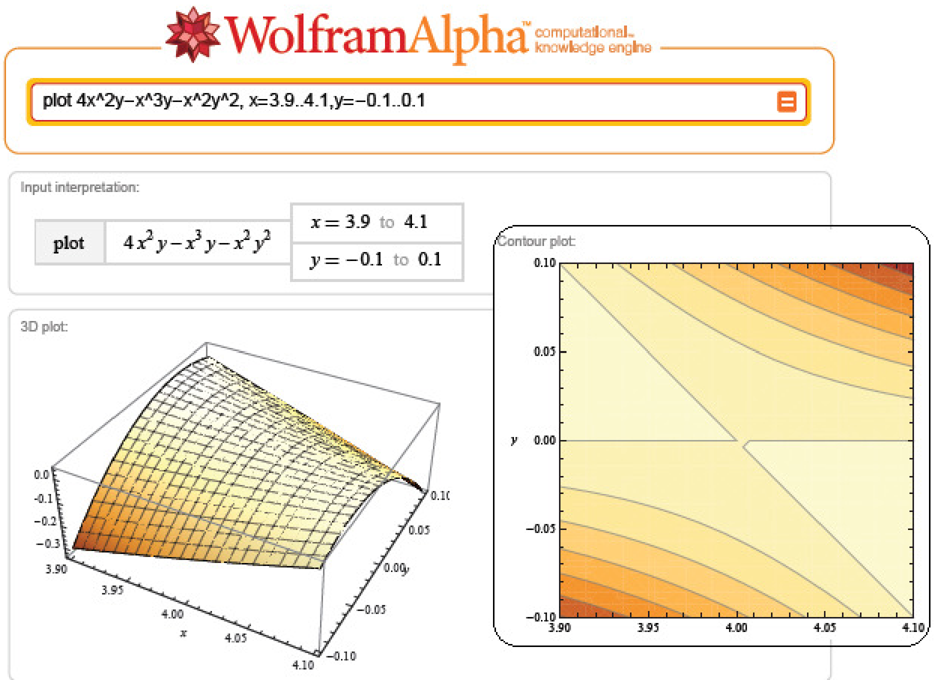
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3D plot:
Contour plot: