

1. Polynomials

Exercise 1a. Input the expression $x^2+5x-14$ and press Enter. Have a look at the results – you can find the graph of a function, other ways of writing the same expression (see: *alternate forms*), the roots of a function, the discriminant (Δ), etc...

Exercise 1b. Try out the command `Roots[x^3-4 x^2+x+6]`.

Exercise 1c. Find roots of the following polynomials:

$$x^3-5x^2-8x+48$$

$$x^4-4x^3-8x^2+36x-9$$

Exercise 2. Try out the following commands:

`Plot[x^3+x^2-x-1]`

`plot x^3+x^2-x-1`

`plot x^3+x^2-x-1,x=-1..1`

`plot x^3+x^2-x-1,x=-2..4`

Exercise 3. Try out the following commands:

`plot x^3+x^2-x-1, x^2-1`

`plot x^3+x^2-x-1,x^2-1, Cos[x]`

`plot Cos[x],1-x^2,x=-2Pi..Pi`

Exercise 4a. Try out the following commands:

`PolynomialQuotient[x^7+1,x^2+1,x]` – gives the result of division of x^7+1 by x^2+1

- x behind the last comma means that it's the main variable in both polynomials

`PolynomialRemainder[x^7+1,x^2+1,x]` – gives the remainder of division of x^7+1 by x^2+1

Exercise 4b. Find the result and the remainder:

$$(x^4+4x^3+6x^2+5x+2)/(x^2+x+1)$$

$$(x^4-2x^3+4x^2-6x+8)/(x-1)$$

2. The absolute value

Exercise 5. Try out the following commands:

`Abs[-5]`

`Abs[E-Pi]`

`plot Abs[x]`

`plot Abs[x^2+5x-14],x=-15..x=10`

`plot Abs[Abs[x]-1]`

`plot Sqrt[x^2]`

3. The $\text{sgn}(x)$ function

Exercise 6. Try out the following commands:

`plot Sgn[x]`

`Sgn[1]+Sgn[-2]`

`plot Sgn[Cos[x]]`

`plot Sgn[x^2-1]`

`plot Sgn[Sgn[x]]`

4. Exponential and logarithmic functions

Exercise 7a. Try out the following commands:

```
plot 2^x, E^x, 3^x, x=-10..4
plot Abs[2^(x-2)-2], x=-10..8
plot E^x
```

Exercise 7b. Check which number is bigger: e^π or π^e ?

Exercise 8. Check which number is the biggest one:

$$2^2, 2^{(2^2)}, 2^{\sqrt{2}}, 2^{\frac{1}{2}}, 2^{(\frac{1}{2})^2}, 2^{(2^{0.5})}, (2^2)^2.$$

Exercise 9a. Try out the following commands:

```
plot Log[x], Log[10,x], x=0..10 - Log[x] means the natural logarithm ln(x)
- use Log[10,x] to acquire the decimal logarithm

Log[2,1/8]
16^(Log[2,3])
```

Exercise 9b. Calculate exact values of the following expressions:

$$\text{a) } \log_{\sqrt{2}} 16, \quad \text{b) } \log_4 0.5, \quad \text{c) } \log_{\frac{2}{3}} 2, 25, \quad \text{d) } \log_{\frac{1}{9}} 3\sqrt[3]{3}, \quad \text{e) } \log_{\frac{1}{3}} \frac{3}{\sqrt[3]{9}}$$

Exercise 9c. Calculate exact values of the following expressions:

$$\text{k) } 3^{\log_6 4 + 2 \log_6 3}, \quad \text{l) } \left(\frac{1}{x}\right)^{\log_x 8}, \quad \text{m) } 10 \cdot 10^{0.5 \log 9 - \log 2}.$$

Exercise 10. Plot the graphs of the following functions:

```
|2 log_4(5x-1)-4|+1
log_10(-ln(x))
log_0.5(2+x)-0.5
```

5. Trigonometric functions

Exercise 11a. Try out the following commands:

```
Sin[Pi/2]
Cos[Pi/3]
Tan[Pi/4]
Cot[Pi/3]
Plot Sin[x], Sin[2x], Sin[0.5x], x=-4Pi..4Pi
```

Exercise 11b. Plot the graphs of the following functions:

$$\text{a) } f(x) = \left| \frac{1}{2} \sin\left(2x - \frac{\pi}{2}\right) - 1 \right|, \quad \text{b) } f(x) = \left| \left| \operatorname{tg}(|x|) - 1 \right| - 1 \right|, \quad \text{c) } f(x) = \left| \left| -\operatorname{ctg}\left(x + \frac{\pi}{4}\right) \right| - 1 \right|,$$

$$\text{d) } f(x) = \left| -\cos\left(|x + \frac{\pi}{2}\right) - \frac{1}{2} \right|.$$

Exercise 12a. Check, that by using the **Simplify[]** command you can acquire very useful formulas:

```
Simplify[Cos[x+y]]
Simplify[Cos[2x]]
Simplify[Sin[x]+Sin[y]]
```

Exercise 12b. Find the formulas for $\cot(2x)$ and $\tan(x+y)$.

Exercise 13. Plot the graphs of the following functions:

a) $y = \sin x + |\sin x|$, b) $y = \operatorname{ctg}\left(\frac{x-|x|}{2}\right)$, c) $y = \operatorname{sgn}(\cos x)$, d) $y = |\sin^4 x - \cos^4 x|$,
 e) $y = \sqrt{1 + \cos 2x}$, f) $y = \operatorname{tg}(\sqrt{x^2})$, g) $y = 2 \sin x |\cos x|$.

6. Cyclometric functions

Exercise 14. Try out the following commands:

```
Asin[1/2]
Acos[Sqrt[2]/2]
Atan[Sqrt[3]]
Acot[Sqrt[3]/3]
plot Asin[x],Acos[x],x=-1..1
plot Atan[x]
```

Exercise 15. Plot the graphs of the following functions:

a) $f(x) = 2 \arcsin\left(\frac{3x+2}{5}\right) + \frac{\pi}{2}$, b) $f(x) = \frac{1}{2} \arccos\left(\frac{x-2}{2}\right) + \frac{\pi}{4}$, c) $f(x) = \pi - 0.5 \operatorname{arctg}(x+1)$,

Exercise 16. Plot the graphs of the following functions:

a) $f(x) = \arcsin(\sin x)$, b) $f(x) = \sin(\arcsin x)$, c) $f(x) = \cos(2 \arcsin x)$.