1. Sequence limits

Exercise 1. Try out the following commands:

limit (1+1/n)^n as n->infinity
limit (4n^5-6n^2+1)/(6n^5+n^3-8) as n->infinity
limit (5*9^n - 5^(n+1)+3)/(-3^(2*n) +2^(3n+1)-1) as n->infinity
limit Sqrt[n]/(Sqrt[n+Sqrt[n+Sqrt[n]]]) as n->infinity

Unfortunately, some limits turn out to be too difficult for Wolfram Alpha:

limit (1/n^2 + 2/n^2 + 3/n^2 + 4/n^2 + ... + (n-1)/n^2) as n->infinity

This limit doesn't exist:

limit Cos[Pi*n]*(n^2+2n)/(2n^2-5n) as n->Infinity

and as a result we obtain: *undefined In the interval…*

Exercise 2. Calculate the following limits:

$$\text{a)}\ \lim_{n\to\infty}\sqrt[n]{e^n+\pi^n+8^n},\qquad \text{b)}\ \lim_{n\to\infty}\sqrt[n]{7+\cos(\pi n)},\qquad \text{c)}\ \lim_{n\to\infty}\frac{5\cdot 4^n+3\cdot\sin(n!)}{2^{2n}+7}$$

Exercise 3. Calculate the limit with a parameter:

Solve[limit a*n^2/(3n^2-n) as n->infinity = 2, a]

2. Function limits

Exercise 4. Try out the following commands:

limit (1-1/x)^x as x->-infinity
limit (6-2Sqrt[x])/(x-9) as x->9
limit Sin[4x]/x as x->0
limit E^(Sin[Acot[(2+x)/(2-x)]]) as x->Infinity

Exercise 5. Calculate function limits:

$$(a)\ \lim_{x\to-\infty}\cos(\operatorname{arctg}\,x)\qquad (b)\ \lim_{x\to\infty}\sin(\operatorname{arctg}\,x)\qquad (c)\ \lim_{x\to\infty}\log_2\left|\frac{x+1}{x^2+2}\right|\qquad (d)\ \lim_{x\to\infty}\arcsin\frac{1-x}{x+1}$$

Exercise 6. Try to calculate a limit that doesn't exist:

limit |x|/x as x->0

As an answer, you obtained two separate one-sided limits (*one-sided limits*).

The following commands compute one-sided limits:

lim E^(1/x) as x->0+
lim E^(1/x) as x->0-

Exercise 7. Check if these limits exist:

$$(a)\ \lim_{x\to2}\frac{\sin(2x-4)}{|x-2|}\qquad (b)\ \lim_{x\to0}\frac{\operatorname{tg}\,|2x|}{3x}\qquad (c)\ \lim_{x\to0}\frac{|x|}{|\sin x|(x+3)}\qquad (d)\ \lim_{x\to3}\frac{(3-x)^3}{|x-3|}$$