1. Computing derivatives

Exercise 1. Try out the following commands:

(x^3 + 4x^2+Cos[x])'
(x^3 + 4x^2+Cos[x])''
(x^3 + 4x^2+Cos[x])'''
(x^x)'

8th derivative of x^10
nth derivative of E^x
nth derivative of 2^x

2nd derivative of x^5
3rd derivative of x^5

Exercise 2. Calculate the derivatives of the following functions:

$(a)\ \ln^3 x$ $(b)\ \ln(1+x^2)$ $(c)\ \sqrt{x^2+3x}$ $(d)\ \sqrt[4]{\cos x}$ $(e)\ \arcsin(\sin x)$

$(f)\ \text{arctg}\ (\text{tg}\ x)$ $(g)\ e^{3x^3+2x}$ $(h)\ e^{\ln x}$ $(i)\ \ln e^x$ $(j)\ \sin(x^2)$

2. Applications of derivatives

Exercise 3. Try out the following commands – they expand the function into the Maclaurin series. Look for the answer in *series representations*:

series E^x, x=0
series Sin[x], x=0

Exercise 4. Expand the functions into Maclaurin's series (in the surrounding of 0): a) **y = cos x**, b) **y = ln(1+x)**.
Expand the function **y=ln x** into Taylor's series in the surrounding of 1.

Exercise 5. Try out the commands that find extremes of the function:

minimize x^3-x
maximize x^3-x

check that these work exactly the same:

min[x^3-x]
max[x^3-x]

Exercise 6. Find extremes of the following functions:

a) $f(x)=\frac{x^2+1}{x}$, b) $f(x)=3x^4-4x^3-6x^2+12x+4$, c) $f(x)=\text{arctg}\ x-\ln(1+x^2)$,

Exercise 7. Try out commands that find the highest and the lowest value on a closed set:

maximize x+2\x - 2,x=1..4
minimize x+2\x - 2,x=1..4

Exercise 8. Find the highest and the lowest values of the following functions on a given set:

$$f(x)=x^2\ln x,\ x\in[\tfrac{1}{e},e], \qquad f(x)=\ln^2 x-\ln x,\ x\in[1,7],$$

Exercise 9. Unfortunately, there are no commands that find monotonicity intervals or convexity intervals, but we can skip this problem by using the following commands:

Solve[(x*E^(-x))'>0, x] - finds an interval in which the function increases
Solve[(x*E^(-x))'<0, x] - finds an interval in which the function decreases
Solve[(x*E^(-x))''>0, x] - finds an interval in which the function is convex
Solve[(x*E^(-x))''<0, x] - finds an interval in which the function is concave

Look for the solution in *Solution*.

Exercise 10. Check monotonicity, convexity and concavity of **$f(x)= -x^4+12x^3-46x^2+60x+1$**.

Exercise 11. Try out the command that finds inflection points:

inflection -x^4+12x^3-46x^2+60x+1

Exercise 12. Find inflection points of the following functions:

$$\text{a) } f(x)=\frac{\ln 2x}{x},\ \text{b) } f(x)=\ln(1+x^2),\ \text{c) } f(x)=x\sin(\ln x).$$

Exercise 13. Try out the command that finds asymptotes:

asymptotes (2x^3 + 4x^2 - 9)/(3 - x^2) - all possible asymptotes
vertical asymptotes (2x^3 + 4x^2 - 9)/(3 - x^2) - only vertical asymptotes
horizontal asymptotes (2x^3 + 4x^2 - 9)/(3 - x^2) - only horizontal asymptotes
oblique asymptotes (2x^3 + 4x^2 - 9)/(3 - x^2) - only oblique asymptotes

Exercise 14. Find all asymptotes of the following functions:

$$\text{a) } f(x)=\frac{x^3}{3(x^2-x-2)},\ \text{b) } f(x)=\frac{3x^4+3x}{5x^2-5x},\ \text{c) } f(x)=\frac{x^2}{2(x-3)},\ \text{d) } f(x)=x\ln\frac{2x}{x-2},\ \text{e) } f(x)=2x-\sin x.$$