Matrices

Exercise 1. Try out the following commands:

{{1},{2},{3}}
{{1,2,3},{4,5,6}} – each row of the matrix must be given in brackets **{ }**

Try out:
{{1,2,0},{3,4,-1}}+2{{5,6,1},{7,8,0}} – addition and multiplication by a number
{{1,2,3},{4,5,6}}.{{2},{7},{6}} – matrices are multiplied using a **dot**, not a „*"
{{2,1},{2,3},{4,5},{7,9}}.{{2,3},{1,1}}
Transpose[{{1,2,3},{4,5,6}}] – matrix transposition

Try raising the matrix to a given power:
{{2,3},{1,1}}^2
{{2,3},{1,1}}.{{2,3},{1,1}}

a) Write the following matrices in Wolfram Alpha:

$$\begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 & 3 & 1 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 5 & 2 & 0 \\ -2 & 0 & 2 & 0 \\ 2 & 3 & 0 & 5 \end{bmatrix}$$

b) For matrices:

$$A = \begin{bmatrix} 2 & 5 & 0 \\ 7 & 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \\ 2 & 5 \end{bmatrix}$$

perform the following calculations:

$$\text{a) } A^T + B, \ \text{b) } 2A - B^T, \ \text{c) } (B^T - A)^T, \ \text{d) } (3A + 2B)^T, \ \text{e) } A^T - B^T.$$

c) For matrices:

$$A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}, C = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

perform the following calculations:

$$\text{a) } A \cdot B, \ \text{b) } B \cdot A, \ \text{c) } A^T \cdot B, \ \text{d) } A \cdot C, \ \text{e) } C^T \cdot B$$

d) Calculate A^2+A^3 for the following matrix:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

Exercise 2. Try out commands calculating a determinant:

Det[{{1,2},{3,4}}]
Det[{{1,2,3},{4,5,6},{0,-1,-2}}]
Try to find a determinant of a non-square matrix:
Det[{{1,2,3},{4,5,6}}]

a) Calculate determinants:

$$\text{a)} \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix}, \text{b)} \begin{vmatrix} 1 & 4 \\ -1 & 1 \end{vmatrix}, \text{c)} \begin{vmatrix} 2 & 6 \\ -4 & 0 \end{vmatrix}, \text{d)} \begin{vmatrix} 2 & 0 & 1 \\ 1 & 1 & 2 \\ -1 & 2 & 1 \end{vmatrix}, \text{e)} \begin{vmatrix} 3 & 1 & -2 \\ 3 & -2 & 4 \\ -3 & 5 & -1 \end{vmatrix}$$

b) Try out:
(Det[{{x,1,1},{1,x,1},{1,1,x}}]) > 0 – look for the solution in the *Solutions* area

By analogy, solve the following inequalities with respect to x:

$$\text{a)} \ \det\left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + x \cdot \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right) \leq 0, \quad \text{b)} \begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & x & 1 \\ 0 & 2 & 0 \\ 1 & x & x \end{vmatrix} > x^2.$$

Exercise 3. Try out commands that find the inverse of a matrix:

Inverse[{{1,2},{3,4}}]
Inverse[{{1,0},{0,0}}] – this matrix is not invertible!

a) Invert matrices:

$$\text{a)} \begin{bmatrix} 3 & -2 \\ 5 & 4 \end{bmatrix}, \quad \text{b)} \begin{bmatrix} -1 & 2 \\ -5 & 6 \end{bmatrix}, \quad \text{c)} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 0 \\ 1 & -1 & 1 \end{bmatrix}, \quad \text{d)} \begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & 1 \\ 2 & 1 & 1 \end{bmatrix},$$

b) To solve this matrix equation:

$$\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \cdot A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

we need to compute:

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

This requires the use of the following instruction: **Inverse[{{1,2},{-1,0}}].{{1,1},{1,1}}**

Solve matrix equations (find matrix X):

a) $AX = B, \qquad A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & 1 \\ 2 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 0 & 3 \\ 1 & 1 & 0 \end{bmatrix},$

b) $XA = B, \qquad A = \begin{bmatrix} 3 & -2 \\ 5 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 2 \\ -5 & 6 \end{bmatrix},$

c) $XA = B, \qquad A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix},$

Exercise 4. Try out commands that solve systems of equations:

$$\begin{cases} x - 2y + 3z = 1 \\ 2x - y + 5z = 1 \\ 3x - 4y + 8z = 3 \end{cases}$$

Solve[{{1,-2,3},{2,-1,5},{3,-4,8}}.{x,y,z}=={1,1,3},{x,y,z}]

$$\begin{cases} 5x + 4y - 2z + 3u = 2 \\ 3x + 2y - 6z + 9u = 4 \\ 4x + 3y - 4z + 6u = 3 \end{cases}$$

Solve[{{5,4,-2,3},{3,2,-6,9},{4,3,-4,6}}.{x,y,z,u}=={2,4,3},{x,y,z,u}]

In general:
A – the coefficient matrix
B – constant terms
X={x,y,z,….} - variables
Solve[A.X==B,X] solves the system

Solve these systems of equations:

$$a)\ \begin{cases} 2x + y - z + t = 1 \\ y + 3z - 3t = 1 \\ x + y + z - t = 1 \end{cases},$$

$$b)\ \begin{cases} x + 2y + z + t = 7 \\ 2x - y - z + 4t = 2 \\ 5x + 5y + 2z + 7t = 1 \end{cases},$$

$$c)\ \begin{cases} x + 2y + 3z + t = 1 \\ 2x + 4y - z + 2t = 2 \\ 3x + 6y + 10z + 3t = 3 \\ x + y + z + t = 0 \end{cases},$$

$$d)\ \begin{cases} 2x + y + z = 1 \\ 3x - y + 3z = 2 \\ x + y + z = 0 \\ x - y + z = 1 \end{cases}.$$

Exercise 5. Try out the command finding the matrix rank:

MatrixRank[{{1,-1,2},{3,-1,5},{1,-1,2},{-2,2,3}}]

Find the ranks of these matrices:

$$\begin{bmatrix} -2 & 3 \\ -4 & 6 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix}, \begin{bmatrix} 4 & -8 & -4 & 12 & 18 \\ 3 & -6 & -3 & 9 & 12 \end{bmatrix}, \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix},$$

Exercise 6. Try out commands finding the eigenvalues of a matrix:

EigenValues[{{1,6},{5,2}}]
EigenValues[{{1,2,3},{4,5,6},{7,8,9}}]

Find the eigenvalues of the following matrices:

$$\begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -4 \\ 0 & 4 & -3 \end{bmatrix}, \quad \begin{bmatrix} 1 & 3 & 1 \\ 2 & -2 & 0 \\ 1 & 0 & 3 \end{bmatrix}.$$