

## Complex numbers

**Exercise 1.** Try out the following commands:

**Sqrt[3]/2 + 2I**  
**(1-I)(2+3I)**  
**1/(3-5I)**  
**1/3 - 5I**  
**(2+3I)^10**

Remember to always write the imaginary unit **I** with a capital letter (just as other important constants: **E** and **Pi**).

**Calculate:**

$$\begin{array}{ll} \text{a) } (-3 - 2i) - (-3 + 5i), & \text{c) } \left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^3, \\ \text{b) } (4 + 2i) \cdot (5 - i), & \text{d) } \frac{2i}{1-i} - \frac{2i}{1+i}, \end{array}$$

**Exercise 2.** Try out the following commands:

**Abs[4+2I]** – the modulus of a complex number  
**Re[4+2I]** – the real part of a complex number  
**Im[4+2I]** – the imaginary part of a complex number  
**Conjugate[4+2I]** – the conjugate of a complex number  
**Arg[1+I]** – the angle between the OX axis and the vector corresponding to the complex number

**Calculate for given z:**

$$\begin{array}{l} \text{a) } \operatorname{Im}\left(\frac{1}{z}\right) + \operatorname{Re}(z), \quad z = 1 - 3i, \\ \text{b) } \operatorname{Im}\left(\frac{1}{z}\right) + \frac{2|z|}{\operatorname{Re}(z)}, \quad z = 4 - 3i, \\ \text{c) } \operatorname{Im}((iz)^2), \quad z = 2 + i. \end{array}$$

**Exercise 3.** Try out the following commands:

**Solve[z^2+z+1==0]** finds complex roots of  $z^2+z+1=0$   
**Solve[z\*Conjugate[z]+2z-2I=0]**  $z \cdot \bar{z} + 2z + 2i = 0$   
**Solve[z^3+z^2+z+1==0]**  $z^3+z^2+z+1=0$   
**Solve[Re[z^2-2]=Conjugate[z]]**  $\operatorname{Re}(z^2-2) = \bar{z}$   
**Solve[z^2==-1]** solves  $z^2 = -1$ , which means finding square complex roots of **-1**!  
**Solve[z^3==-1]** cubic complex roots of **-1**  
**Solve[z^3==I]** cubic complex roots of **I**

**a) Solve these complex equations:**

$$\begin{array}{ll} \text{a) } z^2 + 3\bar{z} = 0, & \text{d) } (z + \bar{z}) + i(z - \bar{z}) = 2i - 6, \\ \text{b) } 2z + (1 + i)\bar{z} = 1 - 3i, & \text{e) } (i - 3)z = 5 + i - z, \\ \text{c) } \frac{z+1}{\bar{z}-1} = -1, & \text{f) } \frac{1-3i}{3z+2i} = \frac{2i-3}{5-2iz}, \end{array}$$

**b) Find all complex roots:**

$$\text{a) } \sqrt[3]{-8}, \quad \text{b) } \sqrt{1-i}, \quad \text{c) } \sqrt[6]{1}, \quad \text{d) } \sqrt{\sqrt{3}+i}, \quad \text{e) } \sqrt{-4+4\sqrt{3}i},$$