Single integrals – revision from the 2nd semester

Exercise 1. Try out the following commands:

integrate 2x^5+6x+10/x
integrate 2E^x(1-E^(-x)/x)
integrate 1/((Cos[x])^2(Sin[x])^2)
integrate Cot[x]
integrate 1/(x^2+x+1) - *notice, that instead of* ***Atan*** *we got* ***tan^{-1}****. That's not a mistake, that's just notation you should get used to.*
integrate 1/(x(Log[x]+1))
integrate x*Atan[x]
integrate E^xCos[x]
integrate 1/((x-1)(x-5))
integrate 1/(x^3+1)
integrate (2-Sin[x])/(2+Cos[x])
integrate 1/(x^(1/3)+x^(1/2))

Compute integrals:

$$\text{a) } \int \sqrt{2x+1}dx, \qquad \text{b) } \int x\sqrt{x^2+7}dx, \qquad \text{c) } \int \frac{x^2}{3\sqrt[3]{x+2}}dx,$$

Exercise 2. Try out the following commands:

integrate x^2, x=2..4
integrate Sin[x], x=-Pi/2..Pi – *the area under the OX axis is marked with pink color – it was subtracted from the area over the OX axis (marked with the blue color).*
integrate E^x, x=-Infinity..0
integrate 1/x, x=0..Infinity – *this integral is divergent*

Compute integrals:

$$\text{a) } \int_2^e \frac{1}{x-1}dx, \qquad \text{b) } \int_1^9 x\sqrt{x}dx, \qquad \text{c) } \int_0^3 \frac{x}{x^2+1}dx, \qquad \text{d) } \int_{\frac{\pi}{2}}^{\pi} \cos 2x dx,$$

Double integrals

Exercise 3. Try out the following commands:

double integral (1-2x^2y), x=0..1,y=-2..1
double integral (E^(x+y)), x=-2..1,y=-1..1

Compute integrals:

$$\text{(a) } \iint_D \frac{x}{y^2}dxdy, \text{ where } D = [1,2] \times [4,6],$$

$$\text{(b) } \iint_D \frac{x^2}{1+y^2}dxdy, \text{ where } D = [0,2] \times [0,1],$$

$$\text{(c) } \iint_D x^2 + \frac{1}{1+y^2}dxdy, \text{ where } D = [0,2] \times [0,1],$$

$$\text{(d) } \iint_D (2x - 3y^2)dxdy, \text{ where } D = [-1,1] \times [0,2],$$

Exercise 4. Try out the following commands:

double integral (x^2y), x=1..2,y=1/x..Sqrt[x]
double integral (x+2y),x=1..4,y=-2Sqrt[x]..-Sqrt[x]
double integral (6-2x-3y), x=0..3, y=0..-2x/3+2

Compute integrals:

(a) $\iint_D x^2 y dxdy$, where D: $y = -\sqrt{x}$, $y = -2$, $x \geq 0$,

(b) $\iint_D xy dxdy$, where D: $y = -x^2 + 4$, $y = 3\sqrt{x}$, $y = 0$,

(c) $\iint_D (x^2 + y) dxdy$, where D: $y = x^2$, $y = \sqrt{x}$,

(d) $\iint_D \frac{x^2}{y^2} dxdy$, where D: $y = \frac{1}{x}$, $y = x$, $x = 2$,

Triple integrals

Exercise 5. Try out the following commands:

integrate (x+y*z),x=0..1,y=0..2,z=-0.5..1
integrate (x^2+2yz+z^3x),x=0..1,y=0..2,z=0..0.5

Compute integrals:

(a) $\iiint_R (xy \sin z) dxdydz$, $R = [\frac{1}{6}, \frac{1}{2}] \times [0, 1] \times [0, \pi]$,

(b) $\iiint_R (\frac{y}{1+x^2} - z \cos y) dxdydz$, $R = [-1, 1] \times [0, \frac{\pi}{2}] \times [-2, 3]$,

(c) $\iiint_R (z 2^{x-y}) dxdydz$, $R = [0, 1] \times [0, 1] \times [-2, 1]$.

Exercise 6. Try out the following commands:

integrate (x+y),x=-1..1,y=x^2..1,z=0..2
integrate (x*y),x=0..3,y=0..-x+3,z=0..3-x-y
integrate r, r=0..4, y=0..2Pi,z=r..4

Calculate the integral of a function *x+y* **in the solid S, knowing that S is a tetrahedron bounded by a plane** *3x+6y+9z=1* **and by the planes of the coordinate system.**