

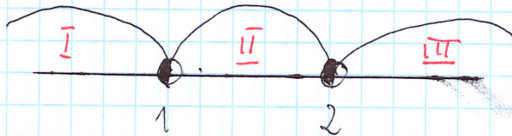
What to say in front of the blackboard - a brief tutorial

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Solve

$$|x-1| + |x-2| > 7x$$

\downarrow \downarrow
 $x_0=1$ $x_0=2$



I $x \in (-\infty, 1)$

$x-1$ (-) \rightarrow a sign must be changed
 $x-2$ (-) \rightarrow be changed

$$\begin{aligned} -(x-1) - (x-2) &> 7x \\ -x+1 - x+2 &> 7x \\ -2x+3 &> 7x \\ -9x &> -3 \quad / : (-9) \\ x &< \frac{1}{3} \\ x &\in \underline{(-\infty, 1/3)} \end{aligned}$$

II $x \in (1, 2)$

$x-1$ (+)
 $x-2$ (-) a sign must be changed

$$\begin{aligned} (x-1) - (x-2) &> 7x \\ x-1 - x+2 &> 7x \\ 1 &> 7x \\ \begin{cases} x < 1/7 \\ x \in (1, 2) \end{cases} & \Rightarrow x \in \emptyset \quad \downarrow \text{no answers} \end{aligned}$$

III $x \in (2, \infty)$

$x-1$ (+)
 $x-2$ (+)

$$\begin{aligned} x-1+x-2 &> 7x \\ 2x-3 &> 7x \\ -5x &> 3 \\ \begin{cases} x < -3/5 \\ x \in (2, \infty) \end{cases} & \Rightarrow x \in \emptyset \quad \downarrow \text{no answers} \end{aligned}$$

the absolute value of x minus 1, plus the absolute value of x minus 2 is greater than 7 times x

Firstly, both expressions in the modulus bar must be compared to 0, then I get two values: 1 and 2.

Secondly I draw the OX axis and I mark on it values I got in the previous step. On OX axis I create three intervals with half-empty and half-full circles at 1 and 2.

My next step is considering the first interval. In this case x belongs to the interval from minus infinity to 1 (inclusive). After rewriting two expressions from the modulus bars I choose any number between minus infinity and 1, for example 0. I put 0 instead of x and then I see that both $(x-2)$ and $(x-1)$ are negative, so I rewrite terms on the left-hand side with a changed sign. I solve the inequality and I get that x is smaller than $1/3$ in the interval I.

Now I have to check the second interval. In this interval x belongs from 1 to 2 (inclusive). I have to check again the sign of $(x-1)$ and $(x-2)$. I have to choose a number between 1 and 2. Let's say I choose $1\frac{1}{2}$. After I put $1\frac{1}{2}$ instead of x I see that I have to rewrite $(x-1)$ with the same sign and $(x-2)$ with a changed sign. After solving the inequality I got a contradiction which means that there are no solutions in this interval.

In the third interval x is greater than 2. I do all the steps with choosing a number from the interval and writing it instead of an x again. In this case I choose, for example, 4 and I put it into $(x-1)$ and $(x-2)$, I get two pluses. Finally I get a contradiction again.

The best, final step is taking answers from all intervals and we get $x \in (-\infty, 1/3)$