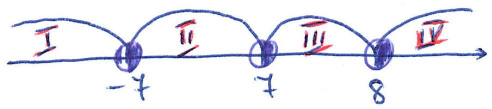
Agnieszka Malinowska 1st year EPM student

$$|x-7|+|x+7|+|8-x|=x-8$$

First, we find the roots of all the expressions in modulus bars. There are: 7, -7, 8, because $x-7=0 \Rightarrow x=7$, $x+7=0 \Rightarrow x=-7$, $8-x=0 \Rightarrow 8=x$

We draw an axis on which we write the roots. We divide an axis into intervals (I always remember to do the intervals with closed brackets on the left side of the interval, but you can do it as you wish)



We write the first interval and solve the equation for it. That means that we write that x belongs to the interval from minus infinity to minus seven (open brackets) and we establish the sign of every expression in modulus bars for the x belonging to the first interval.

For example, for $x \in (-\infty,-7)$ we can choose eg. -10. For -10 the first expression in modulus bars $(x-7)$ is negative, so we put a minus before it. The second expression is also negative - we change the sign before the expression, the third expression is positive so it stays as it is (we don't change the sign)

I $x \in (-\infty,-7)$

$x-7$ ⊖

$x+7$ ⊖

$8-x$ ⊕

$$-(x-7)-(x+7)+8-x=x-8$$

$$-x\cancel{+7}-x\cancel{-7}+8-x=x-8$$

$$-3x+8=x-8$$

$$-4x=-16$$

$$x=4$$

$x \in \emptyset$

As we solve it, x is equal to four. We compare the solution with the interval and we see, that the interval doesn't cover 4. x cannot be 4 for this interval, so it's contradiction. x belongs to an empty set for this interval.

$$x \in \emptyset$$

Analogously we cope with other intervals. We write down the interval, we check the sign of every expression in modulus bars by taking one number from interval as an example, we write down whole equation with changed signs. We solve it and compare with the interval and we write the answer for the interval.

II $x \in \langle -7, 7)$

$x-7$ ⊖
$x+7$ ⊕
$8-x$ ⊕

$$-(x-7)+x+7+8-x=x-8$$
$$\cancel{-x}+7+\cancel{x}+7+8-x=x-8$$
$$22-x=x-8$$
$$-2x=-30$$
$$x=15 \rightarrow x \in \emptyset$$

x equal to 15, doesn't belong to the second interval. x belongs to an empty set.

III $x \in \langle 7, 8)$

$x-7$ ⊕
$x+7$ ⊕
$8-x$ ⊕

$$x\cancel{-7}+\cancel{x}\cancel{+7}+8\cancel{-x}=x-8$$
$$x+8=x-8$$
$$0=16$$

it is false that 0 equals 16
it is contradiction
x belongs to an empty set

$x \in \emptyset$

IV $x \in \langle 8, \infty)$

$x-7$ ⊕
$x+7$ ⊕
$8-x$ ⊖

$$x\cancel{-7}+x\cancel{+7}-(8-x)=x-8$$
$$2x-8+x=x-8 \quad /+8$$
$$3x=x \quad /-x$$
$$2x=0$$
$$x=0$$

compare with the 4th interval, x = 0 doesn't belong to the fourth interval. x belongs to an empty set, $x \in \emptyset$

Now we take the solutions of all intervals. We see that for every interval $x \in \emptyset$.
So there is no solution for this equation. $\boxed{x \in \emptyset}$