What to say in front of the blackboard - a brief tutorial

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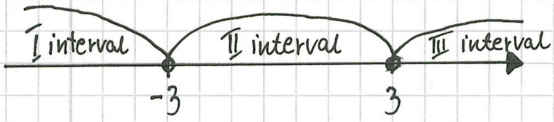
Solve: $|2x+6| + |x-3| = 2x$

$2x = -6 \;/:2$
$x = -3$

$x - 3 = 0$
$x = 3$



The absolute value of $2x$ plus 6 plus the absolute value of x minus 3 is equal to $2x$.

First of all, I compare both expression in the modulus bars to zero and we get two roots: -3 and 3

Then we draw the Ox-axis, on which we mark the numbers we obtained in the previous step. We create 3 intervals and draw circles at -3 and 3 - they are half-empty and half-full. (You can choose which part of the circle is empty and which is full).

I interval $x \in (-\infty; -3\rangle$

$2x+6$ sign $\ominus$, because $2\cdot(-10)+6 = \ominus 14$
after changed sign we get: $-2x-6$

$x-3$ sign $\ominus$, because $-10-3 = \ominus 13$
after changed sign we get: $-x+3$

and $x-3$ is also negative

Let's consider the first interval - here x belongs to the interval from minus infinity ($-\infty$) to minus 3 (inclusive - means a closed bracket).
Then we rewrite two expressions from the modulus bars to check what sign I should put in front of these expressions. Then I choose any number between minus infinity and minus 3. I chose minus 10. If I put -10 instead of x, then $2x+6$ is negative. So I rewrite the equation in the following way: both expressions have to be written with change sign. Next I have to solve an equation.

$-2x-6-x+3 = 2x$
$-2x-x-2x = -3+6$
$-5x = 3 \;/:(-5)$
$x = -\frac{3}{5}$ $\quad x \in \emptyset$

x is equal to $-\frac{3}{5}$, so it doesn't belong to the interval I (so x belongs to the empty set)
In the II interval x belongs to -3 to 3 (inclusive).
Now we check again the signs for interval II.
I choose a number, let's say 1. For $2x+6$ is positive and for $x-3$ it's negative. So I have to change sign for the second expression. And I solve equation again. After calculating x is equal to 9 so it doesn't belong to interval II, so x belongs to empty set $\emptyset$

II interval $x \in (-3; 3\rangle$

$2x+6$ sign $\oplus$, because $2\cdot 1+6 = \oplus 8$

$x-3$ sign $\ominus$, because $1-3 = \ominus 2$
after changed sign we get: $-x+3$

$2x+6-x+3 = 2x$
$-x = -9 \;/:(-1)$
$x = 9$
$x \in \emptyset$

Analogously in the last interval (III) x belong from 3 to infinity (two open brackets). Again we choose 1 number from III interval. Let's say 5. If I substitute it for x I get two pluses. Finally x is equal to -3 so it does not belong to III interval, so the set is empty again.

III interval $x \in (3; +\infty)$

$2(5)+6 = \oplus 16$
$5-3 = \oplus 2$
no changes signs

$2x+6+x-3 = 2x$
$x = -3$
$x \in \emptyset$

Finally, taking answers from all intervals: There is no answer. Every x belongs to the empty set.
$x \in \emptyset$