

What to say in front of the blackboard

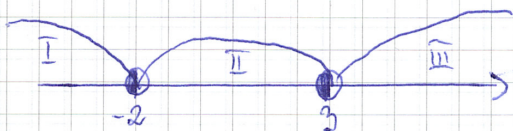
- a brief tutorial.

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SOLVE

$$|x-3| - |2x+4| < x$$

$\begin{matrix} 3 & & -2 \\ | & & | \end{matrix}$



The absolute value of x minus 3, minus the absolute value of two x plus 4 is smaller than x .

Firstly, I compare both expressions in the modulus bars to 0, and I get two values 3 and -2.

I draw the OX axis, on which I marked the numbers -2 and 3, which are x -intercepts. I create three intervals and draw circles at -2 and 3, which are half-empty and half-full.

Let's start calculating with first interval - here, x belongs to the interval from minus infinity to minus 2 (inclusive). I rewrite my two expressions from the modulus bars. Then, I choose any number from interval, for example minus 10. If I put minus 10 instead of x , then both $(x-3)$ and $(2x+4)$ are negative.

So now, I rewrite the inequality in the following way: $(x-3)$ and also $(2x+4)$ are written with changed sign. Now I have my inequality solved.

x belongs to the interval from minus infinity to minus 2 (inclusive).

So now, once again I have to check the sign of $(x-3)$ and $(2x+4)$. For example I choose 0. For x equal 0, I receive one minus and one plus.

I have my second inequality solved, x belongs to the interval from minus one quarter to three (inclusive).

In the last interval x is greater than 3. I choose a number which is greater than 3, let's say 10. If I put 10 instead of x I get two pluses. But now I received a contradiction! I have no solution in this interval.

So final result is $x \in (-\infty, -2] \cup (-\frac{1}{4}, 3]$.

I $x \in (-\infty, -2]$

$$\begin{aligned} x-3 & \ominus \\ 2x+4 & \ominus \end{aligned} \rightarrow \text{a changed sign}$$

$$\begin{aligned} -(x-3) + (2x+4) & < x \\ -x+3+2x+4 & < x \\ x+7 & < x \\ 0 & < -7 \end{aligned}$$

$0 < -7$ - always true in this interval

$x \in (-\infty, -2]$

II $x \in (-2, 3]$

$$\begin{aligned} x-3 & \ominus \\ 2x+4 & \oplus \end{aligned} \rightarrow \text{a changed sign}$$

$$\begin{aligned} -(x-3) - (2x+4) & < x \\ +3 - x - 2x - 4 & < x \\ -3x - 1 & < x \\ -4x & < 1 \\ x & > -\frac{1}{4} \end{aligned}$$

$x \in (-\frac{1}{4}, 3]$

III $x \in (3, +\infty)$

$$\begin{aligned} x-3 & \oplus \\ 2x+4 & \oplus \end{aligned}$$

$$\begin{aligned} x-3 - (2x+4) & < x \\ -x-7 & < x \\ -2x & < 7 \\ x & < -\frac{7}{2} \end{aligned}$$

$x \in \emptyset$