What to say in front of the blackboard - a brief tutorial

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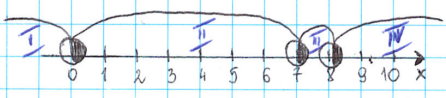
Solve:

$$|7-x| - |8-x| + |x| = 10$$

$x_0 = 7$ $x_0 = 8$ $x_0 = 0$

the absolute value of 7 minus x, minus the absolute value of 8 minus x, plus the absolute value of x is equal to 1

first of all, I compare all the expresions in the modulus bars to 0 and I get three values: 7, 8 and 0



then, I draw the OX axis, on which I mark the numbers I obtained in the previus step. I create four intervals and draw circles at 7, 8 and 0 (they are half-empty and half-full)

now, I consider all intervals.

In the first interval x belongs to from minus infinity to 0 (open bracket). Then I rewrite three expressions from the modulus bars and I choose any number which belongs to the interval, for example -2 If I put -2 instead of x, then (7-x) and (8-x) are positive and (x) is negative so I rewrite the quality in the following way (7-x) and (8-x) are written in the same sign and (x) is written with a changed sign. Now I solve the quality.

I. $x \in (-\infty; 0)$

$7-x$ ⊕
$8-x$ ⊕
x ⊖

$$(7-x) - (8-x) + (-x) = 10$$
$$7 - x - 8 + x - x = 10$$
$$-x = 11$$
$$x = -11$$

x belongs to the first interval so we get the first solution of the quality which is $x = -1$

In the second interval x belongs from 0 (closed bracket) to 7 (open bracket). In the same way, I chose a number belongs to this interval, check the sign and solve.

II. $x \in \langle 0; 7)$

$7-x$ ⊕ (for example $x = 4$)
$8-x$ ⊕
x ⊕

$$(7-x) - (8-x) + (x) = 10$$
$$7 - x - 8 + x + x = 10$$
$$x = 11 \notin \langle 0, 7)$$

↯ contradiction

$x \in \emptyset$

if x doesn't belong to the consider interval I get a contradiction, so there are no solutions in this interval

I do the same with third and fourth interval.

III. $x \in \langle 7; 8)$

$7-x$ ⊖ (for example $x = 7\frac{1}{2}$)
$8-x$ ⊕
x ⊕

$$-(7-x) - (8-x) + x = 10$$
$$-7 + x - 8 + x + x = 10$$
$$3x = 25$$
$$x = 25/3 \notin \langle 7, 8)$$

$x \in \emptyset$ ↯ contradiction

X doesn't belong to the considered interval so there is no solution

IV. $x \in (8; +\infty)$ (for example $x = 9$)

$4 - x$ (−)
$8 - x$ (−)
$8\ x$ (+)

$$-(4-x)-[-(8-x)]+x=10$$
$$-4+x-[-8+x]+x=10$$
$$-4+x+8-x+x=10$$
$$x=9$$

x belongs to the consider interval and I get the second solution of my quality which is $x = 9$

Finally, taking answers from all intervals I get two solutions of my quality which are:

$x = -11$ and $x = 9$