What to say in front of the blackboard - a brief tutorial

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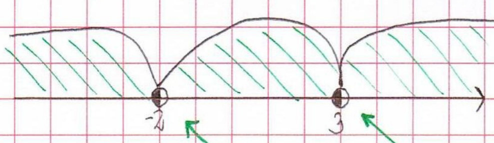
Solve

$$|x-3|+|x+2|>5$$

$x_0=3 \qquad x_0=-2$



I $x \in (-\infty,-2]$

$x-3$ −
$x+2$ −

$$-(x-3)-(x+2)>5$$
$$-x+3-x-2>5$$
$$-2x>4 \quad /:(-2)$$

$$x \in (-\infty,-2] \Leftarrow \begin{cases} x<-2 \\ x \in (-\infty,-2] \end{cases} \cap$$

II $x \in (-2,3]$

$x-3$ −
$x+2$ +

$$-(x-3)+(x+2)>5$$
$$-\cancel{x}+3+\cancel{x}+2>5$$
$$5>5 \quad ↯$$

III $x \in (3,+\infty)$

$x-3$ +
$x+2$ +

$$+(x-3)+(x+2)>5$$
$$\cancel{x}-3+\cancel{x}+2>5$$
$$2x>6$$

$$x \in (3,+\infty) \Leftarrow \begin{cases} x>3 \\ x>3 \end{cases} \cap$$

1) The absolute value of x minus 3 plus the absolute value of x plus 2 is greater than 5

2) First of all, I compare both expressions in modulus bars to 0, and I get two values 3 and -2

3) I draw the Ox axis on which I mark the numbers I obtained in the previous step. I create three intervals, and draw circles at 3 and minus 2 - they are half-empty and half-full.

4) Let's consider the first interval - here x belongs to interval from minus infinity to minus 2 (inclusive). I rewrite two expressions from the modulus bars. Then I choose any number between minus infinity and minus 2 - let's say I choose minus 4. If I put minus 4 instead of x, then (x-3) is negative and (x+2) is negative either, so I rewrite the inequality in the following way: (x-3) and (x+2) are written with a changed sign. Now I solve the inequality and x is smaller than minus 2

5) Now I check signs again like I have done before, but this time have no answer (contradiction)

6) My last interval is from three to infinity. Once again I check the signs in both cases I got plus so I rewrite all inequality without changing anything but without modulus bars, after solving my inequality I got three smaller than x so it belongs to my interval.

My final result is:

$$x \in (-\infty,-2) \cup (3,+\infty)$$