

What to say in front of the blackboard - a brief tutorial.

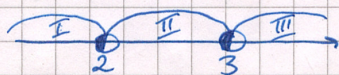
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Solve:

$$|2-x| - |3-x| = 4+x$$

$$\downarrow \quad \quad \downarrow$$

$$x_0 = 2 \quad x_0 = 3$$



I $x \in (-\infty, 2)$

$$\begin{array}{l} 2-x \oplus \\ 3-x \oplus \end{array}$$

$$(2-x) - (3-x) = 4+x$$

$$2-x-3+x = 4+x$$

$$x = -5$$

II $x \in (2, 3)$

$$\begin{array}{l} 2-x \ominus \\ 3-x \oplus \end{array} \leftarrow \text{a changed sign}$$

$$-(2-x) - (3-x) = 4+x$$

$$-2+x-3+x = 4+x$$

$$x = 9 \quad \leftarrow \text{contradiction}$$

III $x \in (3, +\infty)$

$$\begin{array}{l} 2-x \ominus \\ 3-x \ominus \end{array} \leftarrow \text{both signs changed!}$$

$$-(2-x) + (3-x) = 4+x$$

The absolute value of 2 minus x minus the absolute value of 3 minus x is equal to 4 plus x.

At the beginning, I compare both expressions in the modulus bars to zero and I get two roots: 2 and 3.

Next, I draw the ox-axis, on which I mark the numbers I obtained in a previous step. I create 3 intervals and draw circles at numbers 2 and 3 - they are half-full and half-empty (I can choose which part is half-full and which is half-empty).

Now I consider all intervals. In the first interval x belongs from minus infinity to 2 (closed bracket). Then I rewrite two expressions from the modulus bars and I choose any number which belongs to the interval, for example 1. If I put 1 instead of x, then (2-x) and (3-x) are positive so I rewrite expression without changing their signs.

x belongs to the first interval, so I get the first solution of the quality which is x=-5.

In the second interval x belongs from 2 (open bracket) to 3 (closed bracket). In the same way I choose a number that belongs to this interval, check the sign and solve.

If x doesn't belong to the considered interval I get a contradiction, so there are no solutions in this interval.

I do the same with the third interval.

$$-2+x+3-x = 4+x \quad \quad \quad x = -3 \quad \leftarrow \text{contradiction}$$

Finally, I get 1 solution of my quality which is x = -5.