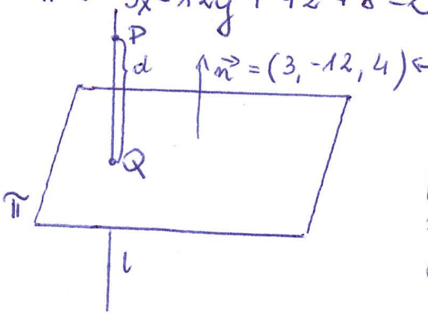
DISTANCE BETWEEN POINT AND PLANE

Calculate the distance between $P = (1, 0, -5)$ and plane π: $3x - 12y + 4z + 8 = 0$



normal vector of plane π from the equation π: $3x - 12y + 4z + 8 = 0$

$l \perp \pi$
$P \in l$
$Q \in l$ and $Q \in \pi$
$d = |\overrightarrow{PQ}|$ ← the length of vector $\overrightarrow{PQ}$

1) At first we have to find a point, that belongs to plane π, ex. point Q

2) Now we have to write the parametric equation of line l

$$l: \begin{cases} x = \overbrace{1}^{\text{point}} + 3s \\ y = 0 - 12s \\ z = -5 + 4s \end{cases}$$

3) Then we should write the equation $\pi(l)$, using the data from the parametric equation of line l

$$\pi(l): 3\underbrace{(1+3s)}_{x} - 12\underbrace{(0-12s)}_{y} + 4\underbrace{(-5+4s)}_{z} + 8 = 0$$

⟶ from the equation of line l

$$3 + 9s + 144s - 20 + 16s + 8 = 0$$
$$s = \frac{9}{169}$$

4) Now we have to put the result (s) into the parametric equation, and we have: (we obtain the coordinates of point Q)

$$l: \begin{cases} x = 1 \cdot \frac{27}{169} \\ y = -\frac{108}{169} \\ z = -5 + \frac{36}{169} \end{cases}$$

5) Then we calculate the coordinates of vector $\overrightarrow{PQ}$ and the length of this vector

$$\overrightarrow{PQ} = \overrightarrow{(1, 0, -5)\left(1\frac{27}{169}, \frac{-108}{169}, -5+\frac{36}{169}\right)} = \left(\frac{27}{169}, -\frac{108}{169}, \frac{36}{169}\right)$$

$$|\overrightarrow{PQ}| = \sqrt{\frac{3^6}{169^2} + \frac{3^6 \cdot 16}{169^2} + \frac{16 \cdot 3^4}{169^2}} = \sqrt{\frac{3^4(3^2 + 3^2 \cdot 16 + 16)}{169}} = \frac{3^2 \cdot \sqrt{9 + 144 + 16}}{169} = \frac{9\sqrt{169}}{169} = \frac{9 \cdot \cancel{13}}{\cancel{169}_{13}} = \frac{9}{13}$$

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