

What to say in front of the blackboard - a brief tutorial

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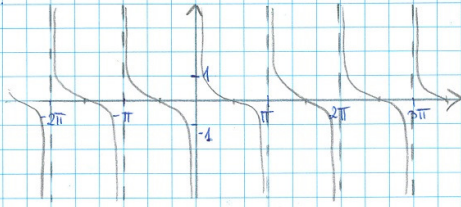
Exercise: Draw the graph of $y = |-\cot(x + \frac{\pi}{4})| - 1$ step by step. Establish the domain and codomain of each function.

STEP 1

I start with $f_1(x) = \cot(x)$

$$D_1: \mathbb{R} \setminus \{k\pi\} \quad k \in \mathbb{Z}$$

$$O_1: \mathbb{R}$$



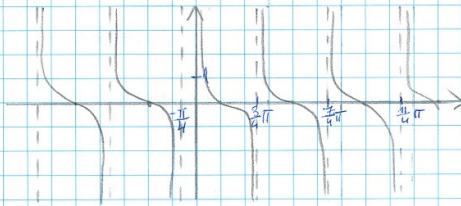
STEP 2

Now, let $f_2(x) = f_1(x + \frac{\pi}{4}) = \cot(x + \frac{\pi}{4})$

$$D_2: \mathbb{R} \setminus \{k\pi + \frac{\pi}{4}\} \quad k \in \mathbb{Z}$$

$$O_2: \mathbb{R}$$

I move my previous graph by $\frac{\pi}{4}$ units to the left.



STEP 3

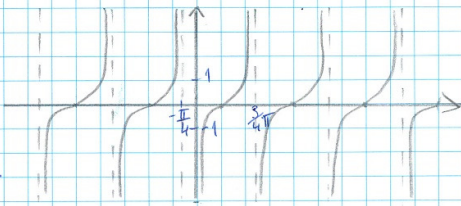
In my next move I create function

$$f_3(x) = f_2(x) = -\cot(x + \frac{\pi}{4})$$

$$D_3: \mathbb{R} \setminus \{k\pi + \frac{\pi}{4}\}$$

$$O_3: \mathbb{R}$$

I turn my previous graph upside down.



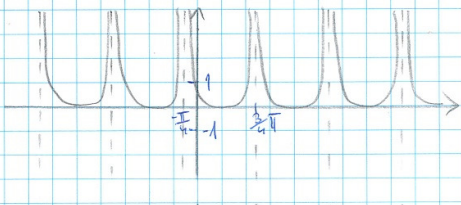
STEP 4

$$f_4(x) = |f_3(x)| = |-\cot(x + \frac{\pi}{4})|$$

$$D_4: \mathbb{R} \setminus \{k\pi + \frac{\pi}{4}\}$$

$$O_4: [0, +\infty)$$

Everything that was below the x-axis is reflected up.



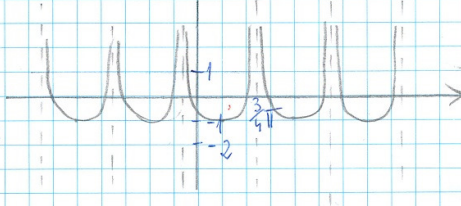
STEP 5

$$f_5(x) = f_4(x) - 1 = |-\cot(x + \frac{\pi}{4})| - 1$$

$$D_5: \mathbb{R} \setminus \{k\pi + \frac{\pi}{4}\}$$

$$O_5: [-1, +\infty)$$

I move my previous graph down by 1 unit.



STEP 6

$$f_6(x) = f_5(x)$$

$$D_6: \mathbb{R} \setminus \{k\pi + \frac{\pi}{4}\}$$

$$O_6: [0, +\infty)$$

Everything that was below the x-axis is reflected up.



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Exercise: Draw the graph of $y = \frac{1}{2}(2\sin(x - \frac{\pi}{4}) - 1)$ step by step. Establish the domain and codomain of each function.

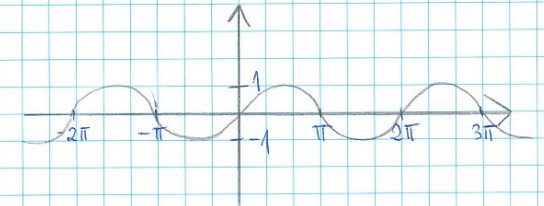
$$\frac{1}{2}(2\sin(x - \frac{\pi}{4}) - 1) = \sin(x - \frac{\pi}{4}) - \frac{1}{2}$$

STEP 1

I start with $f_1(x) = \sin(x)$

$$D_1: \mathbb{R}$$

$$O_1: [-1, 1]$$



STEP 2

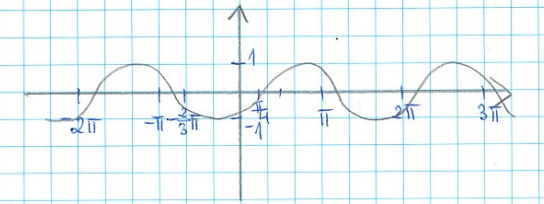
In my next move I create a function

$$f_2(x) = f_1(x - \frac{\pi}{4}) = \sin(x - \frac{\pi}{4})$$

$$D_2: \mathbb{R}$$

$$O_2: [-1, 1]$$

I move my previous function by $\frac{\pi}{4}$ units to the right.



STEP 3

Now, let $f_3(x) = f_2(x) - \frac{1}{2} = \sin(x - \frac{\pi}{4}) - \frac{1}{2}$

$$D_3: \mathbb{R}$$

$$O_3: [-\frac{3}{2}, \frac{1}{2}]$$

I move my previous function down by $\frac{1}{2}$ unit down.

